FIRE: A Fundamental Identification and Replacement Method for Accurate Spectral Test Without Requiring Coherency

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***Abstract*—Achieving coherent sampling is one of the major bottlenecks to perform ADC spectral test, especially when high-precision instruments or high-performance frequency synthesizes are not readily available. If coherent sampling is not achieved, there could be huge leakage in the spectrum, which might lead to inaccurate test results. In this paper, a new fundamental identification and replacement method is presented that can completely eliminate the need for coherent sampling in spectral testing. A two-step fundamental identification method is used to very accurately estimate the noncoherent fundamental. Extensive simulation results show the functionality and robustness of the method. Measurement results obtained in industry labs using commercially available high resolution ADCs successfully validate the proposed method for both accuracy and robustness.**

***Index Terms*—ADC test, DFT, fundamental identification, Newton method, noncoherent sampling, spectral test.**

I. Introduction

ANALOG to digital converters (ADCs) are widely used integrated circuits that have applications in complex circuits, such as system on chips (SoCs). It is very important to test ADCs accurately to guarantee specified performance of a system. ADCs are usually tested for static specifications, such as integral nonlinearity (INL) and differential nonlinearity (DNL) and dynamic specifications, such as total harmonic distortion (THD) and signal to noise ratio (SNR) [1], [2]. Spectral testing of ADCs is also called AC testing and includes testing of ADCs dynamic (frequency dependent) specifications. In contrast, full spectrum testing not only tests dynamic specifications but also focuses on testing all spectral bins including harmonic and nonharmonic bins. Being able to perform full spectrum testing is especially important for systems whose SFDR is limited by nonharmonic spurious tones, such as time-interleaved ADCs. The test setup for both spectral test and Full spectrum test is the same.

Fig. 1 shows a conventional test setup for ADC spectral test. To accurately perform spectral testing, the IEEE Standard for Digitizing Waveform Recorders (IEEE Std. 1057) [3] and IEEE Standard for Terminology and Test Methods for Analog-to-Digital Converters (IEEE Std. 1241) [4] recommend the test setup to satisfy the following five conditions. Firstly, the spectral purity of the input signal to ADC should be about 3 to 4 bits more pure than the ADC under test. That is, to test an N-bit ADC, the input signal should be more than $N+3$ bits pure. The second condition is that the peak-to-peak voltage of the input signal should be slightly lower than the ADC input range so that the output of the ADC is not clipped. The third condition is to have very low relative jitter between the clock and input signals. The fourth condition is that, if possible, the input signal be coherently sampled. Finally, the total number of sampled points (or data record length) should be sufficiently large. The first four conditions mentioned above are very challenging to achieve, and it is important to satisfy all conditions to perform accurate spectral test.

Looking into the future, there is a strong drive to design circuits that have built-in self-test (BIST) capability to decrease the test cost. The area required by the testing circuitry should be very small compared to that of the device under test (DUT). In such circuits, it is impossible to achieve coherent sampling with a self-contained oscillator as signal source implemented on a very small area. Another case is, during characterization of an ADC, the spectral characteristics of the ADC at various input frequencies need to be tested. It would take more test time to achieve coherent sampling in such cases as the frequencies of input signal and clock signal needs to be tuned for each input frequency separately to achieve coherent sampling. This tuning increases the test cost and the product delivery time. In both the above mentioned cases, it is either impossible or more time consuming to achieve coherent sampling. So, there is a strong need to develop new low cost test methods that can eliminate the condition of coherent sampling and still provide accurate spectral results.

Two of the methods that are widely used in both industry and academics are the windowing technique [5]–[10] and the four parameter sine fitting technique [4]. To obtain accurate spectral results with windows, the spectral power of secondary lobes of selected window should be lower than the noise power of the ADC under test. This requires prior knowledge about the type of window to be used to accurately test the ADC. The spectral results obtained are window dependent. Also, for large noncoherent sampling and high resolution ADCs,

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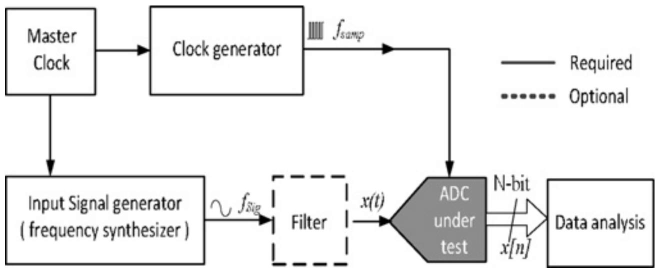

Fig. 1. Spectral test setup for ADC under test.

not all windows can achieve accurate spectral results [11]. The four parameter sine fitting method is used to characterize ADCs and digital oscilloscopes for THD and ENOB [12]–[14]. The method gives accurate values of THD, SNDR, and ENOB. Also, when the harmonic component determines SFDR, accurate value of SFDR is obtained. However, when a nonharmonic component determines SFDR, the method cannot provide accurate value (As shown in Fig. 4 below and as discussed in Section II). In such cases, a method intended for full spectrum test is required. Furthermore, computational efficiency is one of the concerns when data record length is large. In [15], a multisine fitting algorithm was proposed to accurately estimate the fundamental and harmonics of the signal. The method accurately estimates the values of THD, SNR, and ENOB if the three initial frequency estimates are sufficiently close to the true value. However, it cannot provide accurate value of SFDR when a nonharmonic component is responsible.

In the recent past, several methods have been proposed to relax the condition of coherency for spectral testing [16]. A 2-D FFT method was introduced in [17] with a time complexity of O ($M^2 \log^2 M$), where M is the total data record length. A singular value decomposition method was proposed in [18], which involves a time complexity of O (M^3). In [19], a filter bank method was reported that results in an increase in testing circuitry area. A resampling technique was presented in [20], which again results in increasing area due to additional decimator used. In [21]–[25], interpolating DFT (IpDFT) methods were used to eliminate the requirement of coherency. However, such methods cannot provide accurate value of SFDR when a nonharmonic spur dominates the harmonics. In [26], a fundamental identification and replacement method was proposed that can accurately estimate the spectral characteristics. However, the method is not robust to signal frequencies that are close to Nyquist range.

All the above methods suffer from one or more of the issues, such as large computation time, increase in area, lack of robustness across the Nyquist range, dependency of results on the type of window chosen, or the inability to perform full spectrum test. So, it is required to develop a test method that can address all the above issues and accurately perform spectral test without requiring coherent sampling.

In [11], a fundamental identification and replacement method was proposed that is robust over any level of noncoherency. The method provides accurate spectral results for ADCs with medium resolution. However, for very high resolution ADCs, the estimated spectral parameters have errors as the accuracy with which the fundamental was identified was not sufficient.

In this paper, a new fundamental identification and replacement (FIRE) method that addresses all the above issues and performs accurate spectral testing is presented. Compared to the method in [11], a new two-step fundamental identification method that can very accurately estimate the fundamental is proposed. The initial estimates of parameters are obtained in step 1 using closed form expressions. In step 2, Newton method is used to accurately estimate all the parameters. The estimation is done using the frequency domain data and is computationally efficient. The estimated noncoherent fundamental is later removed from the initial data to obtain the residue. A fundamental that is coherently sampled is added to the residue and DFT is performed on the final data to obtain accurate spectral results. The functionality and robustness of proposed FIRE method is verified using both simulation and measurement data.

The remainder of the paper is arranged as follows. Section II discusses ADC spectral test and introduces the problem of noncoherent sampling. The detailed description of proposed FIRE method is provided in Section III. Section IV presents the simulation results and Section V validates the FIRE method using measurement data. Section VI concludes the paper.

II. ADC Spectral Test And Non-coherent Sampling

Let f_{Sig} be the frequency of input signal, f_{Samp} be the clock frequency, M be the total number of data points recorded to measure the spectral characteristics, and J be the total number of periods of the input signal sampled in the recorded data. The four parameters are related by the equation

$$J = M \frac{f_{Sig}}{f_{Samp}}. \tag{1}$$

The sampling is said to be coherent if J in (1) is an integer that is coprime with M and noncoherent if J is a noninteger. In addition, it is recommended that $J > 5$ [4].

A. ADC Spectral Test Using Coherent Sampling

It is recommended to perform coherent sampling to accurately test an ADC. The test setup for coherent sampling was shown in Fig. 1. Fig. 2 shows the spectrum of an ADC output data when sampled coherently. It can be seen that the spectrum is clean and all the above mentioned spectral parameters can be accurately estimated as explained below.

Let *x(t)* be the time domain representation of the analog signal. The signal is ideally a pure sine wave

$$x(t) = A \cos\left(2\pi f_{Sig} t + \phi\right) \tag{2}$$

where, A is the amplitude and ϕ is the initial phase of *x(t)*.

Let *x[n]* be the analog interpretation of the digital output obtained from the ADC whose gain error and offset have been calibrated. *x[n]* can be represented by

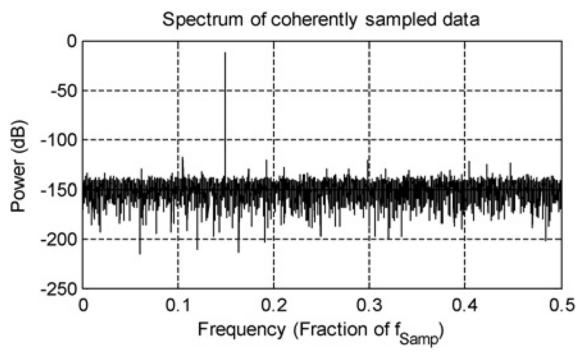

Fig. 2. Power spectrum of coherently sampled ADC output.

$$x[n] = A\cos\left(\frac{2\pi J}{M}n + \phi\right) + \sum_{h=2}^{H} A_h \cos\left(\frac{2\pi h J}{M}n + \phi_h\right) + w[n] \quad (3)$$

for n = 0,1,2, ...,M-1. M is usually selected to be a power of two for faster processing of the FFT algorithm. H is the total number of harmonics present in *x[n]*, A_h and ϕ_h are the amplitude and initial phase of hth harmonic, respectively, such that $A_h << A$ and $\phi_h \epsilon [0, 2\pi)$ for all $2 \le h \le H$. *w[n]* corresponds to white noise in nth sample, which can be due to quantization noise, input referred ADC noise, and additive noise in the input signal. The harmonics in the output of ADC, *x[n],* correspond to the distortion of ADC.

The spectral parameters can be accurately obtained by taking discrete Fourier transform (DFT) of M coherently sampled data points. The DFT of *x[n]* is given by

$$X_k = \frac{1}{M}\sum_{n=0}^{M-1} x[n]e^{-j\frac{2\pi k}{M}n},\ for\ k = 0, 1, ..., M-1 \quad (4)$$

where k represents the frequency bin's index. For example, with coherent sampling, $k = h*J$ represents the frequency bin of the hth harmonic and if $h = 1$, $k = J$ represents the frequency bin of the fundamental. X_0 corresponds to the DC component in signal *x[n]*. Other values of k correspond to noise.

From (3) and (4), neglecting the effect of noise, X_k can be simplified and given as

$$\begin{aligned} X_k = \Bigg(& \frac{A}{2M}\left\{\frac{\sin(\pi(J-k))}{\sin\left(\frac{\pi(J-k)}{M}\right)}e^{j(a(J-k)+\phi)} \right. \\ & \left. + \frac{\sin(\pi(J+k))}{\sin\left(\frac{\pi(J+k)}{M}\right)}e^{-j(a(J+k)+\phi)}\right\} \\ & + \frac{A_h}{2M}\left\{\frac{\sin(\pi(hJ-k))}{\sin\left(\frac{\pi(hJ-k)}{M}\right)}e^{j(a(hJ-k)+\phi_h)} \right. \\ & \left. + \frac{\sin(\pi(hJ+k))}{\sin\left(\frac{\pi(hJ+k)}{M}\right)}e^{-j(a(hJ+k)+\phi_h)}\right\}\Bigg). \end{aligned} \quad (5)$$

It can be seen from (5) that, for coherent sampling

$$X_J = \frac{A}{2}e^{j\phi} \text{and} X_{hJ} = \frac{A_h}{2}e^{j\phi_h}. \quad (6)$$

For other values of k, X_k represents the noise as there is no contribution from the fundamental and harmonics on the bins. The power of fundamental, hth harmonic and noise can be accurately estimated as P_1, P_h, and P_{noise}, respectively, using

$$\begin{aligned} P_1 &= 2|X_J|^2 = \frac{A^2}{2} \\ P_h &= 2|X_{hJ}|^2 = \frac{A_h^2}{2} \\ P_{noise} &= \sum_{\substack{k=1 \\ k\neq J,hJ \\ h=2,...,H}}^{M-1} |X_k|^2. \end{aligned} \quad (7)$$

From (7), the spectral parameters such as THD, SNR, and SFDR for a coherently sampled signal can be calculated using

$$\begin{aligned} THD &= \frac{\sum_{h=2}^{H} P_h}{P_1} \\ SNR &= \frac{P_1}{P_{noise}} \\ SFDR &= \frac{P_1}{2 * \max\limits_{\substack{k=1,...,(M/2) \\ k\neq J}} \left(|X_k|^2\right)} \end{aligned} \quad (8)$$

Equations (6)–(8) give accurate values of spectral parameters. Hence, coherent sampling method can be used for full spectrum testing. However, achieving coherent sampling is a very challenging task, because, it is required to use either very high accuracy frequency synthesizers or phase locked loops (PLL). This increases the test cost or test area.

B. Issues With Noncoherent Sampling

If coherent sampling is not achieved, J in (3) is not an integer and as a result, the spectrum may contain severe skirting as shown in Fig. 3. This phenomenon is widely known as spectral leakage. One of the methods that is recommended to tackle the noncoherent sampling issue is four parameter sine wave fitting method. For small amount of noncoherency, this method can give accurate values of THD and SNR. However, occasionally the method fails in estimating the accurate value of SFDR. This could happen when there is a nonharmonic spur in the spectrum that contributes for SFDR. Fig. 4 shows the spectrum of the measured data of a time-interleaved 9-bit ADC clocked at 800 MHz. It can be seen that there is a nonharmonic spur present that contributes to SFDR. Such spurs could result due to several reasons such as timing mismatches or offset mismatch in a time-interleaved ADC, or variations in power supply. It is important to find such nonharmonic spurs to accurately estimate SFDR. As a result, four parameter sine wave fitting method cannot be used to perform full spectrum test. The same reason holds for using the interpolated DFT methods to test noncoherently sampled data [21]–[25].

Also, if the sampling is not coherent, (6) is no longer valid. Let $J = J_{int} + \delta$, where J_{int} represents the integer part of J and δ represents the non-integer part of J. δ varies from -0.5 to 0.5. Substituting J in (5) gives (9).

It can be seen that unlike in (5), the contribution from fundamental and harmonics on to other frequency bins is no longer zero in (9). As a result, using (6)–(8) to test spectral

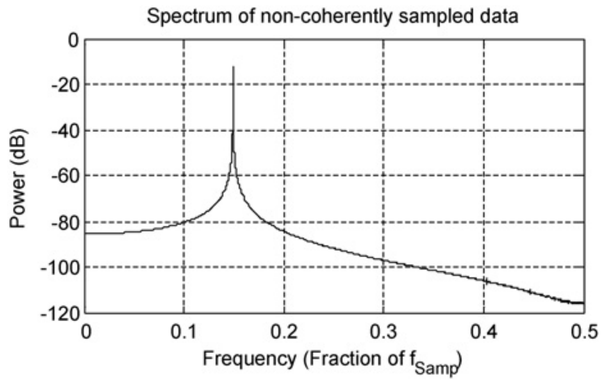

Fig. 3. Power spectrum of noncoherently sampled ADC output.

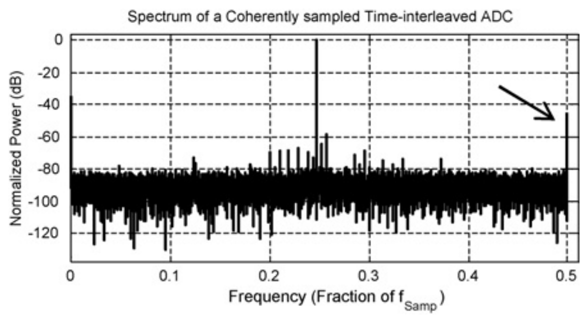

Fig. 4. Power spectrum of coherently sampled time-interleaved ADC showing a nonharmonic spur.

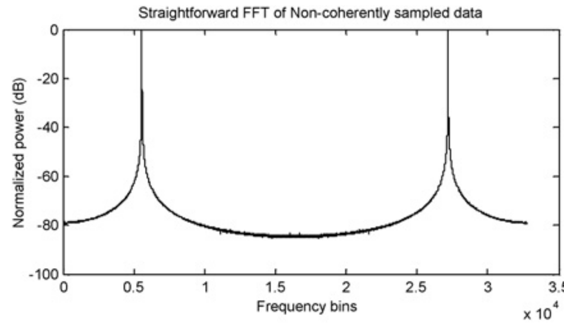

Fig. 5. Spectrum of FFT of noncoherently sampled data showing leakage.

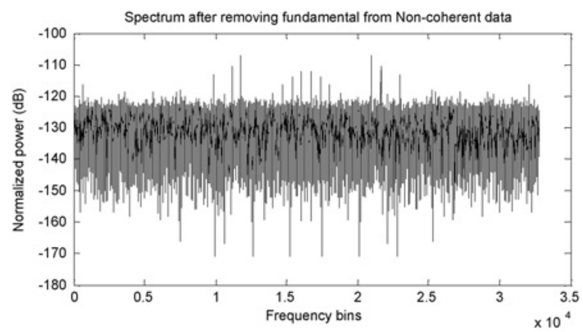

Fig. 6. Spectrum of residue obtained after removing the fundamental.

characteristics of a noncoherently sampled data results in inaccurate values. Such cases of noncoherent sampling are very common and it is important to design a test method that can perform full spectrum test in spite of having noncoherent sampling.

$$X_k = \left(\frac{A}{2M} \left\{ \frac{\sin(\pi(J_{\text{int}} - k + \delta))}{\sin\left(\frac{\pi(J_{\text{int}}-k+\delta)}{M}\right)} e^{j(a(J_{\text{int}}-k+\delta)+\phi)} + \frac{\sin(\pi(J_{\text{int}} + k + \delta))}{\sin\left(\frac{\pi(J_{\text{int}}+k+\delta)}{M}\right)} e^{-j(a(J_{\text{int}}+k+\delta)+\phi)} \right\} + \sum_{h=2}^{H} \frac{A_h}{2M} \left\{ \frac{\sin(\pi(hJ_{\text{int}} - k + h\delta))}{\sin\left(\frac{\pi(hJ_{\text{int}}-k+h\delta)}{M}\right)} e^{j(a(hJ_{\text{int}}-k+h\delta)+\phi_h)} + \frac{\sin(\pi(hJ_{\text{int}} + k + h\delta))}{\sin\left(\frac{\pi(hJ_{\text{int}}+k+h\delta)}{M}\right)} e^{-j(a(hJ_{\text{int}}+k+h\delta)+\phi_h)} \right\} \right) \tag{9}$$

III. Proposed Fundamental Identification and Replacement (FIRE) Method

In this section, a test method is proposed that can take in noncoherently sampled ADC output and perform full spectrum test. Before describing the method in detail, a brief description about the foundation for fundamental identification and replacement methods [11], [26]–[29] is presented.

It can be said that when DFT is applied on noncoherently sampled data, the leakage in the spectrum is mainly due to the fact that the fundamental component is noncoherently sampled. It can also be stated that for high resolution ADC testing, the leakage from any frequency bin (other than the fundamental) to any other frequency tone is significantly below the total noise power of the ADC. This effect of noncoherent fundamental is shown in Figs. 5 and 6.

Fig. 5 is the spectrum of a noncoherently sampled data. As explained earlier, there is severe spectral leakage around the fundamental. However, it can be seen that if the noncoherent fundamental in this data is identified and removed, accurate information of harmonics and noise can be obtained from the spectrum of residue as shown in Fig. 6. However, it is required to accurately identify the noncoherent fundamental to obtain correct spectral results.

A. Fundamental Identification

Several methods were presented in the past to identify the fundamental in a noncoherently sampled data [21]–[25], [30]. One of the proposed methods is the interpolated discrete Fourier transform (IpDFT). The IpDFT methods start from applying windows on the noncoherently sampled data and later perform interpolation to accurately estimate the fundamental component [21]–[25]. In [25], a criterion to choose the optimal window to obtain accurate spectral characteristics was proposed. However, in the proposed FIRE method, the fundamental is identified from the DFT of noncoherently sampled data without using windows. As a result, the method is not dependent on windows and can accurately estimate the fundamental.

Substituting $J = J_{int} + \delta$ in (3), in order to identify the fundamental, it is required to estimate the values of J_{int}, δ, A and ϕ. In the proposed FIRE method, the fundamental component is identified in a two-step process. First step provides the value of J_{int} and initial estimates of δ, A, and ϕ from the DFT of noncoherently sampled data. The second step obtains accurate estimates of δ, A, and ϕ using Newton method. The procedure to identify the fundamental is explained in detail below.

1) *First Step:* The time domain output data of the ADC, *x[n]* in (3) is converted to frequency domain data by taking the DFT of *x[n],* which is given by X_k in (4). Using DFT coefficients, the value of J_{int} and initial values of δ, A, and ϕ are estimated.

Estimate J_{int} and δ J_{int} is estimated by taking the index of frequency bin in half spectrum that contains the maximum power excluding the DC component.

$$\widehat{J}_{\text{int}} = \arg\ \max_{1 \le k \le M/2} |X_k|. \tag{10}$$

The initial value of δ can be estimated with a three-point calibration method using the DFT coefficients. Using (9), for $k = J_{int}$, X_k represents the DFT coefficient of the fundamental and for $k = J_{int} + 1$ and J_{int}-*1*, X_k represents the DFT coefficients of the adjacent bins on either side of the fundamental bin. For high resolution ADCs, when estimating the fundamental, the effect of harmonics can be neglected. Also, in order to obtain a closed form expression for initial value of δ, the term containing $\mathrm{e}^{-\mathrm{j(a(J+k+\delta)+\phi)}}$ can be neglected compared to the term containing $\mathrm{e}^{\mathrm{j(a(J-k+\delta)+\phi)}}$for $M > 1024$ [31]. The neglected term is later considered in the equations to obtain an accurate estimate of δ in the second step. After neglecting the above mentioned terms, X_k can be given by (11)

$$X_k \approx \frac{A}{2M} e^{j\phi} \frac{1 - e^{j2\pi(J-k)}}{1 - e^{j\frac{2\pi(J-k)}{M}}}. \tag{11}$$

$$\text{Let} Y = e^{j2\pi\delta}. \tag{12}$$

For $k = J_{int}$, $J_{int} + 1$ and J_{int}-*1*, using (11)

$$X_{J_{\text{int}}} = \frac{A}{2M} e^{j\phi} \frac{1 - e^{j2\pi\delta}}{1 - e^{j\frac{2\pi\delta}{M}}} = \frac{A}{2M} e^{j\phi} \frac{1 - Y}{1 - Y^{\frac{1}{M}}} \tag{13}$$

$$X_{J_{\text{int}}+1} = \frac{A}{2M} e^{j\phi} \frac{1 - e^{j2\pi\delta}}{1 - e^{j\frac{2\pi(\delta-1)}{M}}} = \frac{A}{2M} e^{j\phi} \frac{1 - Y}{1 - Y^{\frac{1}{M}} e^{-j\frac{2\pi}{M}}} \tag{14}$$

$$X_{J_{\text{int}}-1} = \frac{A}{2M} e^{j\phi} \frac{1 - e^{j2\pi\delta}}{1 - e^{j\frac{2\pi(\delta+1)}{M}}} = \frac{A}{2M} e^{j\phi} \frac{1 - Y}{1 - Y^{\frac{1}{M}} e^{j\frac{2\pi}{M}}}. \tag{15}$$

The above three equations can be used to solve for Y in terms of X_{Jint}, X_{Jint+1}, and X_{Jint-1}

$$Y^{\frac{1}{M}} = e^{j\frac{2\pi\delta}{M}} = \left(\frac{\frac{X_{J_{\text{int}}}}{X_{J_{\text{int}}+1}} - \frac{X_{J_{\text{int}}}}{X_{J_{\text{int}}-1}}}{\frac{X_{J_{\text{int}}}}{X_{J_{\text{int}}+1}} - \frac{X_{J_{\text{int}}}}{X_{J_{\text{int}}-1}} + e^{j\frac{2\pi}{M}} - e^{-j\frac{2\pi}{M}}} \right). \tag{16}$$

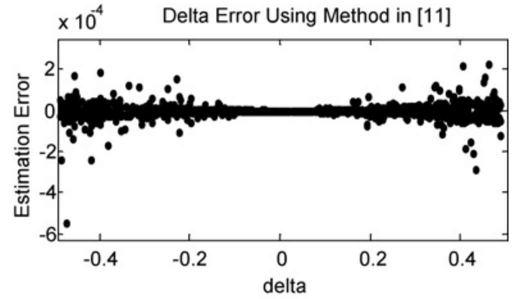

Fig. 7. Error in estimating δ versus actual value of δ using (17).

From (12) and (16), the initial value of δ, δ_0, can be estimated by (17) as

$$\delta_0 = \frac{M}{2\pi} imag \left(\ln \left(\frac{\frac{X_{J_{\text{int}}}}{X_{J_{\text{int}}+1}} - \frac{X_{J_{\text{int}}}}{X_{J_{\text{int}}-1}}}{\frac{X_{J_{\text{int}}}}{X_{J_{\text{int}}+1}} - \frac{X_{J_{\text{int}}}}{X_{J_{\text{int}}-1}} + e^{j\frac{2\pi}{M}} - e^{-j\frac{2\pi}{M}}} \right) \right). \tag{17}$$

Estimate A and ϕ. Now that J_{int} and δ are estimated, the initial values of A and ϕ can be estimated using (13). Taking magnitude of X_{Jint} gives the initial value of A, A_0, as shown in

$$A_0 = 2M \left| X_{J_{\text{int}}} \right| \left| \frac{1 - e^{j\frac{2\pi\delta_0}{M}}}{1 - e^{j2\pi\delta_0}} \right|. \tag{18}$$

The initial value of ϕ, ϕ_0, can be estimated by using A_0 and δ_0 as shown in

$$\phi_0 = -imag \left(\ln \left(\frac{2MX_{J_{\text{int}}}}{A_0} \frac{1 - e^{j\frac{2\pi\delta_0}{M}}}{1 - e^{j2\pi\delta_0}} \right) \right). \tag{19}$$

Hence, using (10), and (17)–(19), J_{int} and the initial values of δ, A, and ϕ are estimated.

2) *Second Step:* It can be noted that (11) involves an assumption to neglect $\mathrm{e}^{-\mathrm{j(a(J+k+\delta)+\phi)}}$term. The error in estimating the value of δ for 1000 runs using this assumption is shown in Fig. 7. It can be seen that the values of 1000 randomly selected δ's were estimated with a maximum error of about 10^{-4}. However, to perform high resolution ADC test, the estimation error should be very small. Also, in order to propose a method that is independent of the resolution of ADC, the error should only be limited by the noise power per bin (i.e., P_{noise}/M). To obtain these requirements, it is necessary to include $\mathrm{e}^{-\mathrm{j(a(J+k+\delta)+\phi)}}$term in estimating the three parameters δ, A, andϕ. The expression of X_{Jint} without neglecting $\mathrm{e}^{-\mathrm{j(a(J+k+\delta)+\phi)}}$term is shown in (20). It can be seen that the expression is a nonlinear equation in δ. Also, X_{Jint} can be represented with both real and imaginary parts given by R_{Jint} and I_{Jint}, respectively. It should be noted that both R_{Jint} and I_{Jint} are functions of J_{int}, A, δ, and ϕ

$$X_{J_{\text{int}}} = \frac{Ae^{j\phi}}{2M} \frac{1 - e^{j2\pi\delta}}{1 - e^{j\frac{2\pi\delta}{M}}} + \frac{Ae^{-j\phi}}{2M} \frac{1 - e^{-j2\pi(2J_{\text{int}}+\delta)}}{1 - e^{-j\frac{2\pi(2J_{\text{int}}+\delta)}{M}}} \tag{20}$$

$$= R_{J_{\text{int}}}(J_{\text{int,}} A, \delta, \phi) + jI_{J_{\text{int}}}(J_{\text{int,}} A, \delta, \phi). \tag{21}$$

Similarly, X_{Jint+1} and X_{Jint-1} need to be modified from (14)–(15) to include the neglected term in (11). It can be seen that two equations are obtained by taking the real part and imaginary part of (21) separately. Doing the same for X_{Jint+1} and X_{Jint-1}, a total of six equations are obtained. Let the six equations be given as $f_1,..,f_6$

$$f_1(J_{\text{int}}, A, \delta, \phi) = R_{J_{\text{int}}}(J_{\text{int}}, A, \delta, \phi) - real(X_{J_{\text{int}}}) \tag{22}$$

$$f_2(J_{\text{int}}, A, \delta, \phi) = I_{J_{\text{int}}}(J_{\text{int}}, A, \delta, \phi) - imag(X_{J_{\text{int}}}) \tag{23}$$

$$f_3(J_{\text{int}}, A, \delta, \phi) = R_{J_{\text{int}}+1}(J_{\text{int}}, A, \delta, \phi) - real(X_{J_{\text{int}}+1}) \tag{24}$$

$$f_4(J_{\text{int}}, A, \delta, \phi) = I_{J_{\text{int}}+1}(J_{\text{int}}, A, \delta, \phi) - imag(X_{J_{\text{int}}+1}) \tag{25}$$

$$f_5(J_{\text{int}}, A, \delta, \phi) = R_{J_{\text{int}}-1}(J_{\text{int}}, A, \delta, \phi) - real(X_{J_{\text{int}}-1}) \tag{26}$$

$$f_6(J_{\text{int}}, A, \delta, \phi) = I_{J_{\text{int}}-1}(J_{\text{int}}, A, \delta, \phi) - imag(X_{J_{\text{int}}-1}). \tag{27}$$

From above six nonlinear equations, the three parameters are accurately estimated by Newton method and least squares. Using Newton method, the value of y in $(k+1)$th iteration, y_{k+1}, is given by

$$y_{k+1} = y_k - B_k \backslash f_k, \tag{28}$$

where "\" operator is the least squares operator, y_k is the vector containing the three estimated parameters in kth iteration, f_k is the vector of $f_1..f_6$ evaluated using estimated values in y_k, and B_k is the Jacobean matrix evaluated using values in y_k as shown below

$$B_k = \left.\begin{bmatrix} \frac{\partial f_1}{\partial A} & \frac{\partial f_1}{\partial \delta} & \frac{\partial f_1}{\partial \phi} \\ . & . & . \\ . & & \\ . & & \\ . & & \\ \frac{\partial f_6}{\partial A} & \frac{\partial f_6}{\partial \delta} & \frac{\partial f_6}{\partial \phi} \end{bmatrix}\right|_{y_k}, \quad f_k = \left.\begin{bmatrix} f_1 \\ . \\ . \\ . \\ . \\ f_6 \end{bmatrix}\right|_{y_k}, \quad y_k = \left.\begin{bmatrix} A \\ \delta \\ \phi \end{bmatrix}\right|_k. \tag{29}$$

It can be mentioned that this method always converges to a global minima as the initial points to start the iterations are very close to the actual values. Also, it can be noticed that the number of operations in each iteration is no longer dependent on the length of the data record. Each iteration involves six equations and three unknowns, thus making the method more computationally efficient compared to other sine fitting methods [12]–[15] that use total data record for each iteration. Using rigorous simulation study, it is seen that a maximum of five iterations would always result in delivering precise values of the three parameters, thus, accurately estimating the fundamental. This accuracy in estimating the three parameters is limited by the noise power per bin (Fig. 9). The error in estimating the same 1000 values of δ's (as in Fig. 7) using this two-step method is shown in Fig. 8. It can be seen that, the estimation error using the two-step method is decreased by three orders (from 10^{-4} to 10^{-7}) compared to that using (17). Also the error obtained using this method is only limited by the noise power per bin as shown in Fig. 9. A total of 50 randomly selected values of δ for each value of SNR are considered and the error in estimating δ is noted down. The data record length (M) for all runs is 4096. With constant

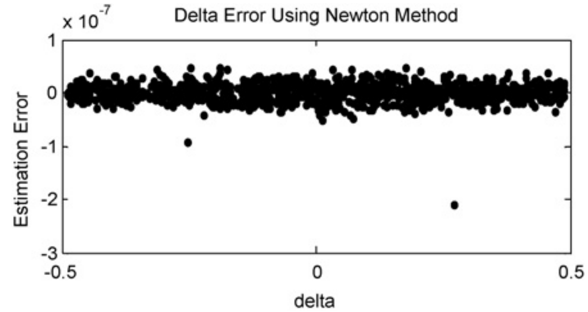

Fig. 8. Error in estimating δ versus actual value of δ using two-step method.

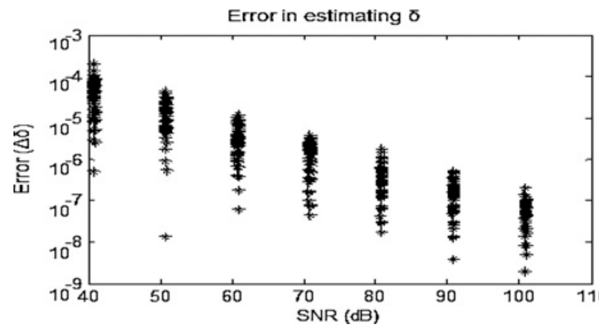

Fig. 9. Error in estimating δ using two-step method for different SNR values. With fixed signal power, as SNR increases, estimation error decreases.

data record length (M) and signal power, as the value of SNR increases, the noise power per bin decreases. It can be seen from Fig. 9 that as the noise power per bin decreases (i.e., as SNR increases), the estimation error also decreases and more accurate values of δ can be obtained. Hence, the proposed two-step fundamental identification method accurately estimates the fundamental component and the accuracy is only limited by the noise power per bin. Let the final estimates of δ, A, and ϕ be given as $\widehat{\delta}$, $\widehat{A}$, and $\widehat{\phi}$ respectively.

B. Estimate Noncoherent Fundamental

Using $\widehat{\delta}$, $\widehat{A}$, and $\widehat{\phi}$, the non-coherent fundamental component in $x[n]$ can be estimated as $x\_nc[n]$ and is given as

$$x\_nc[n] = \widehat{A}\cos\left(\frac{2\pi\left(J_{\text{int}} + \widehat{\delta}\right)}{M}n + \widehat{\phi}\right). \tag{30}$$

C. Construct Coherent Fundamental.

The fundamental component that is coherent (signal corresponding to J_{int} cycles) can be constructed using $\widehat{A}$ and $\widehat{\phi}$, and is given as $x\_c[n]$, as shown in

$$x\_c[n] = \widehat{A}\cos\left(\frac{2\pi\left(J_{\text{int}}\right)}{M}n + \widehat{\phi}\right). \tag{31}$$

D. Fundamental Replacement

Using (30) and (31), the noncoherent fundamental can be removed from the output of ADC and replaced by a coherent fundamental. This replaced output is given as $x_{new}[n]$

$$x_{new}[n] = x[n] - x\_nc[n] + x\_c[n]. \tag{32}$$

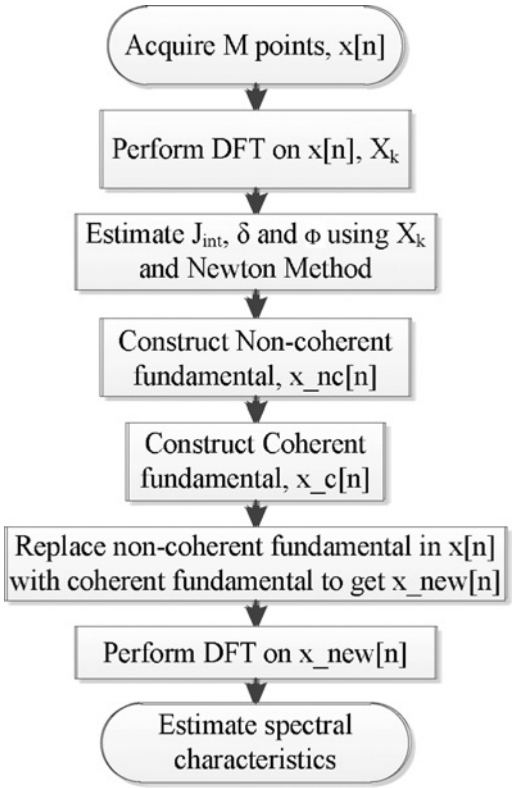

Fig. 10. Flow chart to perform spectral testing using proposed FIRE method.

Since the noncoherent fundamental component in *x[n]* is replaced with a coherent fundamental in $x_{new}[n]$, taking FFT of $x_{new}[n]$ gives accurate spectral results (SNR, SFDR, THD). Thus, the method can be used to perform full spectrum test without using windows and without large increase in area and test time (as shown in simulation results).

The flow chart in Fig. 10 summarizes the steps to be performed for spectral test using FIRE Method.

IV. Simulation Results

In this section, simulation results to verify the functionality and robustness of the proposed FIRE method are presented. The computation time of the proposed method is also compared along with other methods used for noncoherent sampling. In Sections IV and V, one data stream was used to estimate the spectral characteristics. The data record length was selected to accommodate the effect of noise.

A. Functionality

An 18-bit ADC was generated using MATLAB with an INL of 1.2 LSB. The true THD, SFDR, and SNR values of the ADC are obtained by sending a pure sine wave that is coherently sampled. Later, a noncoherently sampled pure sine wave with same amplitude is sent to the same ADC and the proposed method is used to obtain spectral characteristics.

Fig. 11 shows three spectrums of the same ADC obtained using the following cases. The blue spectrum is obtained when the ADC is coherently sampled with $M = 16384$ and $J = 593.00$. The spectrum is clean without any leakage. The other two spectrums are obtained using noncoherently sampled data with $J = 593.1237$. The green spectrum is obtained when DFT is directly performed on the noncoherently sampled data. As expected, there is severe leakage in the spectrum due to noncoherency ($\delta = 0.1237$). However, using proposed FIRE method on the same noncoherently sampled data, the leakage is completely eliminated as shown in the red spectrum. It can also be seen that the red spectrum exactly matches with the blue spectrum. Table 1 lists the spectral results estimated using the proposed method on noncoherently sampled data and the coherently sampled method. It can be seen that the results obtained from noncoherently sampled data using FIRE method are very close to those obtained using coherent sampling method. This shows that the proposed method accurately estimates the spectral characteristics even when an input signal is not coherently sampled.

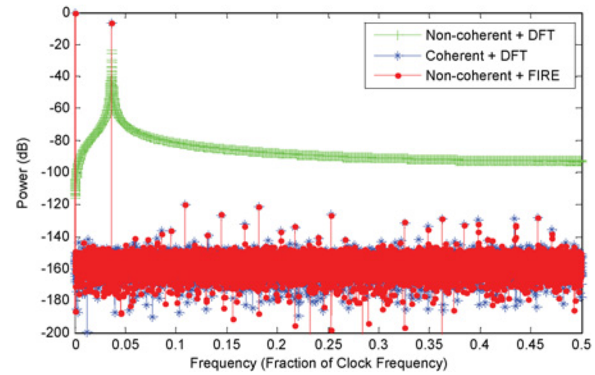

Fig. 11. Plot showing the spectrum of an ADC for three cases. Blue spectrum is obtained using coherent sampling ($J_{int} = 593$), red spectrum is obtained using the proposed FIRE method on noncoherently sampled data ($J = 593.1237$), and green spectrum is obtained after performing DFT on the same noncoherently sampled data (color version of the figure is available at IEEE online or from the authors).

B. Robustness

The robustness of the method with signal frequency and noncoherency is also presented. An 18-bit ADC with INL of 2.4 LSB was simulated. 1000 values of δ and J_{int} corresponding to input signal are randomly generated. The values of δ and J_{int} range from -0.5 to 0.5 (the whole range of δ), and from 0 to $M/2$ (whole Nyquist range), respectively. The THD, SFDR, and SNR of the ADC obtained by coherent sampling are -104.2 dB, 107.5 dB, and 108 dB, respectively. The errors obtained in estimating the THD, SFDR, and SNR values of the ADC with respect to signal frequency (given as a fraction of sampling frequency) are shown in Fig. 12. It can be seen that the values are very accurately estimated and the method is robust for input signal frequency in the whole Nyquist range. Fig. 13 shows the errors in THD, SFDR, and SNR with respect to noncoherency, δ. It can also be seen that the method is robust over the whole range of δ, from -0.5 to 0.5. Hence, the method is robust for input signal frequencies in the whole Nyquist range and for any noncoherency. This robustness study totally eliminates the requirement of coherent sampling for spectral testing using FIRE method.

TABLE I
SPECTRAL RESULTS OF 18-BIT ADC CORRESPONDING TO FIG. 11

Method	THD (dB)	SFDR (dB)	SNR (dB)
Coherent + DFT (Ideal)	−110	113.7	108.5
Non-coherent + FIRE	−109.7	113.6	108.6

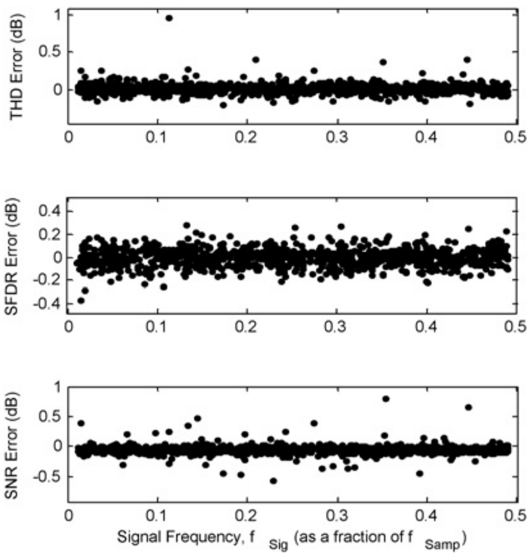

Fig. 12. Error in estimating THD, SFDR, and SNR over the whole range of input signal frequency (from DC to Nyquist range).

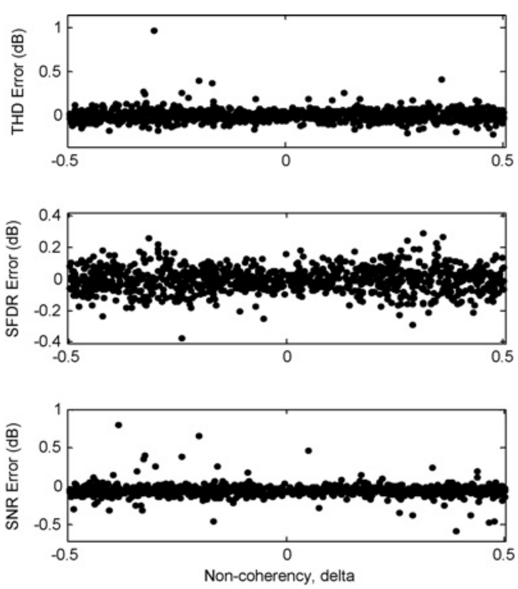

Fig. 13. Error in estimating THD, SFDR, and SNR over the whole range of noncoherency in the fundamental, δ (from −0.5 to 0.5).

C. Computation Time

The calculation time complexity of the proposed method is of the order of $M^* \log_2 M$, since, performing FFT is the only major time consuming block. The time taken by the proposed FIRE method is compared with different windows, the best data record length method [26] and a four parameter sine fitting method [4] in Table 2 (using MATLAB on a 64-bit, Intel Core i5 CPU with 4GB memory). It can be seen

TABLE II
COMPARISON OF CALCULATION TIME
($J = 519.379$, $M = 8192$, $18-$ bit ADC, INL = 1.4LSB)

Method	Time	Functionality
Proposed Method	1.7 ms	Accurate
Best Data Record Method [26]	27.8 ms	Accurate
Window 2 in [32]	2.9 ms	Accurate
Blackman Harris (4-term)	0.7 ms	Inaccurate
Hanning	0.5 ms	Inaccurate
Hamming	0.9 ms	Inaccurate
* Four Parameter Sine Fit (Nonlinear Least Squares)	>25.2 ms	Occasional Inaccurate SFDR

*: Time taken to only estimate the fundamental accurately. Later, 3-parameter fit is required to estimate each harmonic using total 8192 points which results in more computation time (Clause 8.8.1.3 of [4]).

that of all the methods listed, the proposed method provides accurate test results with least computation time. The method in [26] consumes more time as the best data record length is not necessarily a power of two. This results in larger computation time for the FFT algorithm [26]. It can also be seen from the table that only one window can accurately test an 18-bit ADC while the other three windows cannot be used to test. This shows the dependency of results on the type of window used. As a result, prior knowledge about the resolution of ADC is required to perform spectral test using windows. The computation time using a four-parameter sine-wave fitting method using time domain data and nonlinear least squares method is shown. Though the method provides accurate estimates of fundamental, it can be seen that using all time-domain data consumes large computation time. As a result, the proposed FIRE method can be readily used to test any resolution ADC to obtain fast and accurate spectral results.

V. MEASUREMENT RESULTS

In this section, measurement data is used to validate the effectiveness of the proposed FIRE method. Two commercially available ADCs are tested for spectral characteristics with noncoherent sampling using the FIRE method. The first ADC is ADS1282, which is a very high resolution delta-sigma ADC with an SNR of 120 dB. This ADC is used to verify the functionality of FIRE method for very high resolution ADCs. The second ADC is ADS8318, which is a 16-bit, 500 kSPS successive approximation register (SAR) ADC. This ADC is used to verify the robustness of proposed FIRE method with respect to whole range of noncoherency, δ, using measurement data.

A. ADS1282 Test (Functionality Test)

Fig.14 shows the test setup used to test ADS1282. DAC1282 is used to provide the pure input signal to test ADC. Both the DAC and ADC are controlled by the same master clock. The ADC sampling clock frequency is 1 kHz and the input signal frequency for coherent sampling is 31.25 Hz. A total of 4096 points were sampled (M). The value of J obtained is 128 for coherent sampling. With this setup, a clean spectrum is obtained and is given by the blue plot in Fig. 15. Later, the same ADC is noncoherently sampled with signal frequency

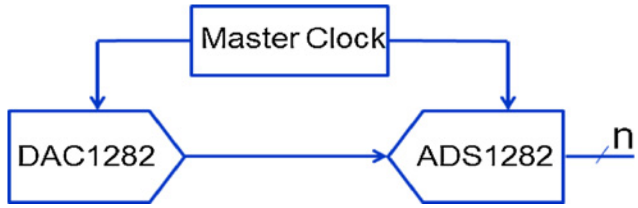

Fig. 14. Test setup for ADS1282 ($f_{Samp} = 1$ kHz, $M = 4096$, $J_{coherent} = 128$, $J_{noncoherent} = 126.781$).

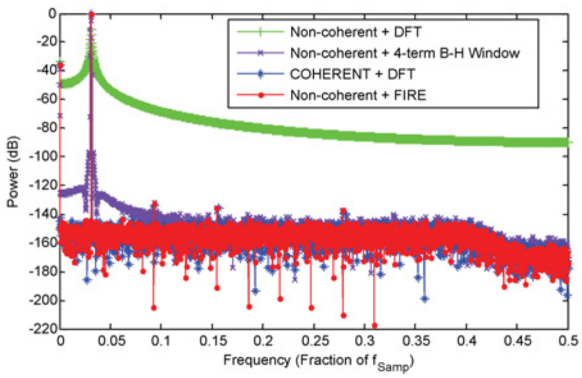

Fig. 15. Plot showing spectrum of ADS1282 for four different cases. The blue spectrum is obtained with coherent sampling ($J = 128$). The red spectrum is obtained when FIRE method is used on noncoherently sampled data ($J = 126.781$). The green spectrum is obtained when DFT is performed on noncoherently sampled data. The purple spectrum is obtained when a four-term Blackman-Harris (B-H) window is used on the noncoherently sampled data (color version of the figure is available at IEEE online or from the authors).

given by 30.952 Hz, which results in J equal to 126.781. This corresponds to noncoherent sampling with δ=-0.219. The spectrum of the output of ADC when FFT is applied on this data is given by the green plot in Fig. 15. As expected there is severe spectral leakage. However, using the proposed FIRE method on this noncoherently sampled data resulted in a clean and accurate spectrum as shown by the red plot in Fig. 15. It can be seen that the red spectrum (FIRE) matches with the blue spectrum (coherent) and provides accurate spectral results for a very high resolution ADC. To show the effect of windows on spectral testing, the spectrum obtained when a four-term Blackman Harris window is used on the same noncoherently sampled data is shown by the purple plot in Fig. 15. Table 3 compares the values of THD, SFDR, and SNR of the ADC using the FIRE method and windows method with the values obtained using coherent sampling method. It can be seen that the FIRE method accurately estimates the parameters. From the purple plot in Fig. 15 and from Table 3, it can be said that four-term Blackman Harris window cannot be used for testing this high resolution ADC. Hence, as mentioned earlier, the choice of window used is dependent on the resolution of ADC.

B. ADS8318 Test (Functionality and Robustness Test)

The second ADC that is tested is ADS8318. The test setup is as shown in Fig. 16. A signal generator is followed by two band pass filters with center frequency at 10 kHz. The output of the filter is fed to the input of ADC. The ADC is clocked at 500 kSPS and a total of 2048 samples were collected. The input signal frequency to achieve coherent sampling is given by 10.009765625 kHz, which gives a value of J equal to 41. Fig. 17 shows the values of THD, SFDR, and SNR along with the spectrum of ADS8318 with coherent sampling. For a value of $\delta = 0.46$, Fig. 18 shows the spectrum of the same ADC using FIRE method. It can again be seen that there is no leakage in the spectrum in spite of noncoherent sampling and the spectral parameters are very accurately estimated.

Later, to test for robustness of the FIRE method for any value of noncoherency (δ) using measurement data, the frequency of input signal is changed from 9.887 kHz to 10.132 kHz so that the value of J varies from 40.5 to 41.5. This covers the whole range of δ from -0.5 to 0.5. The values of THD, SFDR, and SNR are evaluated for each case and plotted in Figs. 19, 20, and 21, respectively. The variation of THD, SFDR, and SNR for different values of δ is expected as only 2048 points are sampled to test a 16 bit ADC. It can be seen that the values of THD, SFDR, and SNR are very accurately estimated using the proposed FIRE method for any value of noncoherency, δ.

Hence, the functionality and robustness of the proposed FIRE method with noncoherent sampling is successfully validated using measurement data from two high resolution ADCs.

TABLE III
SPECTRAL CHARACTERISTICS OF ADS1282 MEASURED USING COHERENT AND NONCOHERENT SAMPLING (IN dB)

METHOD	THD	SFDR	SNR
Coherent	−130.9	133.4	120.3
Non-coherent + FIRE	−129.6	132.3	120.1
Non-coherent + 4-term B-H Window	−126.5	130.8	90.5

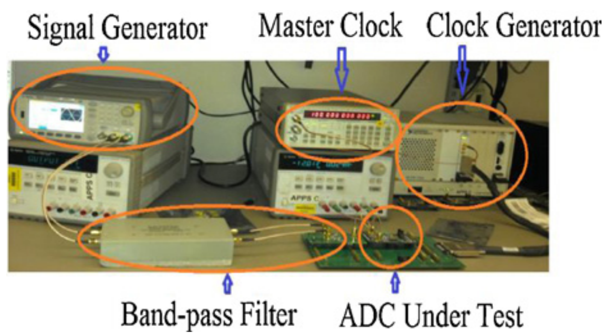

Fig. 16. Test setup for ADS8318.

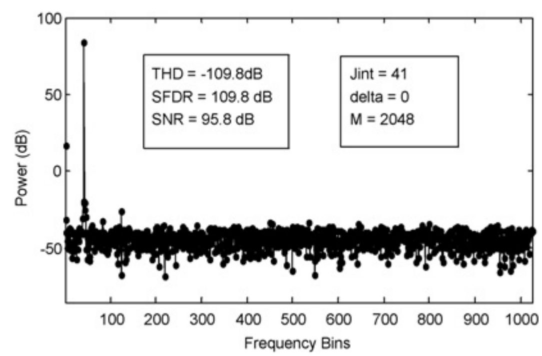

Fig. 17. Spectrum of ADS8318 using coherent sampling.

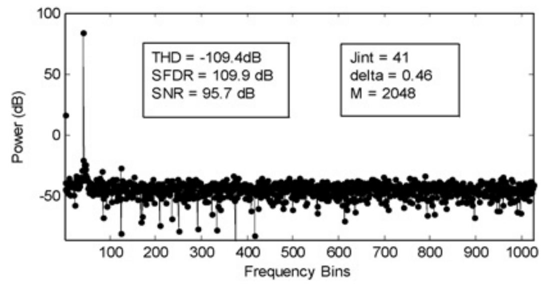

Fig. 18. Spectrum of ADS8318 using non-coherent sampling and proposed FIRE method.

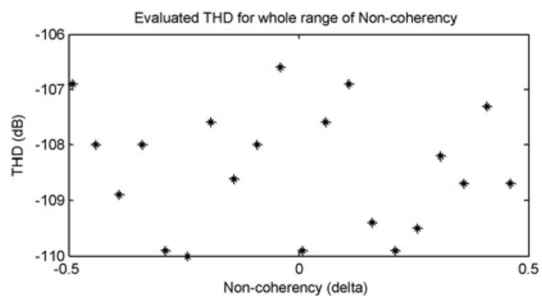

Fig. 19. Measured THD values over whole range of δ for ADS8318.

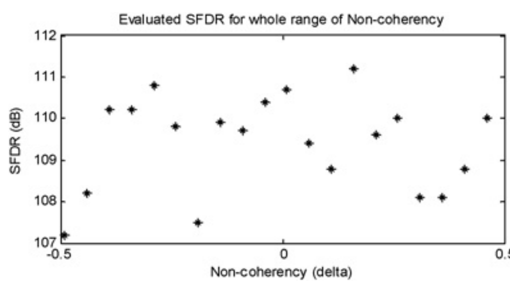

Fig. 20. Measured SFDR values over whole range of δ for ADS8318.

VI. Conclusion

A new Fundamental Identification and Replacement (FIRE) test method was proposed that completely eliminates the requirement of coherent sampling for full spectrum test. A two-step method using DFT and Newton method was described to accurately identify the noncoherent fundamental. It was shown that the accuracy with which the fundamental was identified is only limited by the noise power per bin (P_{noise}/M). As a result, the method can be readily used to test any ADC output without prior knowledge about the resolution of ADC, unlike windowing method. Simulation results were presented to show the functionality and robustness of the proposed FIRE method with respect to any noncoherency (δ) and to any input signal frequency in the whole Nyquist range. The time complexity of the method is of the order of $M^*\log_2(M)$. Thus, all the issues related to previous state-of-the-art techniques, such as large computation time, large area, lack of robustness of the method over the whole Nyquist range, dependency of the results on the window chosen and inability to perform full spectrum test, have been addressed in the proposed FIRE method. Furthermore, measurement results using two commercially available high resolution ADCs were presented that validated

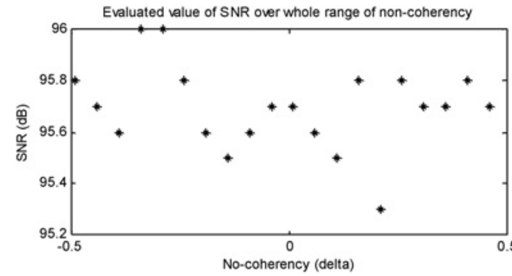

Fig. 21. Measured SNR values over whole range of δ for ADS8318.

the accurate functionality and robustness of the FIRE method. Finally, it can be said that the FIRE method can be used in all forms of test, such as Bench characterization, final test and BIST, to save the cost and effort associated with achieving coherent sampling.

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