A Comparative Study of state-of the-art High Performance Spectral Test Methods

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Abstract

Spectral test of high performance Analog-to-Digital Converters is a very challenging task as the test setup is required to satisfy several stringent conditions. A lot of test time is involved in satisfying these conditions. Several methods have been proposed to relax some of these conditions to decrease test setup time. In this article, four such methods are described and compared based on several criteria related to spectral testing. A summary that could help determine the best method to be used depending on the available test setup is presented.

1. Introduction

Spectral analysis is one of the most widely used methods in several areas such as geophysics, oceanography, medical sciences, etc. It provides information about different frequency components present in a signal that are otherwise not observed in time domain. In electronics, it is extensively used to test dynamic linearity of semiconductor devices such as Analog-to-Digital Converters (ADC). This testing for dynamic parameters using spectral analysis is termed spectral test.

ADCs are one of the most widely used integrated circuits, either as standalone parts or as part of a System-on Chip (SoC). Spectral test measures the dynamic parameters of ADCs and is important for those that are used in high speed applications such as communications [1-2]. In order to accurately perform spectral test, the IEEE standards [3-4] recommend the test setup to satisfy a list of stringent requirements. The test setup that satisfies all the conditions is referred as the '*ideal DFT (Discrete Fourier Transform) based test method*' in this article.

With increase in the resolution and speed of ADCs, the test setup requirements for ideal DFT based test become much more stringent. A lot of effort is spent in satisfying the test setup requirements which increases the test time. Relaxing stringent conditions on test setup not only decreases the test setup time but also facilitates the design of SoCs with on-chip test capability. Several spectral test methods have been proposed that relax one or more of the stringent requirements. With several methods proposed, it is not clear to the user to select a method that provides best results with minimal constraints.

This article aims at providing a comparative study on four methods that relax stringent conditions for spectral test. The methods have been compared for several criteria and a summary on the capability of each method is presented. This enables the user to select a method based on the test setup requirements. The methods for comparison include windowing [5-6], four parameter sine fit [7], Fundamental Identification and REplacemnt (FIRE) [8] and Fundamental Estimation, Removal And Residue Interpolation [9].

2. Ideal DFT based Spectral Testing

Requirements and Challenges

Fig. 1 shows the test setup for ideal DFT based spectral test. The quantitative details of requirements and challenges to satisfy the requirements for ideal DFT based spectral test are as follows.

1) Input Signal: In order to test an N-bit ADC, the input signal should be at least N+3 bits pure. With increase in resolution of ADC, this requirement on input signal is very challenging to satisfy. Filters as shown in Fig. 1 are used occasionally to satisfy this requirement when the input source is not pure enough, thus increasing the test cost.

2) Clock Signal: The contribution from jitter between input and clock signals should be less than the noise floor. Equation (1) is used to get the maximum allowable jitter requirement [10]. Here t_j is rms jitter in the system, f is input frequency. With increase in ADC resolution, the maximum jitter acceptable by the test system needs to be decreased, which increases the cost as more expensive clock generators need to be procured.

$$SNR = 20\log_{10}\left(\frac{1}{2\pi f t_j}\right) \tag{1}$$

3) Sampling: The input signal should be coherently sampled. A signal is said to be coherently sampled if the data record contains an integer number of cycles of the signal. This requires high accuracy signal generators and the required accuracy of signal generators increases with increase in resolution of ADC. In Fig. 1, a master clock is used to control frequencies of both input and clock signals simultaneously to achieve coherent sampling, thus, increasing the test area and cost.

4) Input Amplitude: The peak-to-peak voltage of the input signal should be within the ADC input range. If the standard deviation of noise rms can be given as σ_N and the full scale of ADC as FS, the input signal, V_{IN}, should follow equation (2) to avoid clipping (about 10 times σ_N to be safe from clipping; ADC input range given by [F_b F_t]). In applications such as BIST ADCs, it is required to

consume a lot of area in order to obtain precise amplitude control. This further increases the cost.

$$\left.\begin{aligned}\min\left(V_{IN}\right) > F_b + 10\sigma_N \\ \max\left(V_{IN}\right) < F_t - 10\sigma_N\end{aligned}\right\} \quad (2)$$

5) <u>Data record:</u> The number of samples in data record should be selected such that at least five periods of input signal are sampled [4]. A trade-off is made between required noise floor and data acquisition time.

Working Principle

Single-tone test: Let f_{Sig} be the frequency of input signal, f_{Samp} be the clock frequency, M be the total number of data points recorded to measure the spectral characteristics and J be the total number of periods of input signal in data record. The four parameters are related by equation (3). Let J be given as the sum of J_{int} and δ, where J_{int} is the integer part of J and δ is the non-integer part of J.

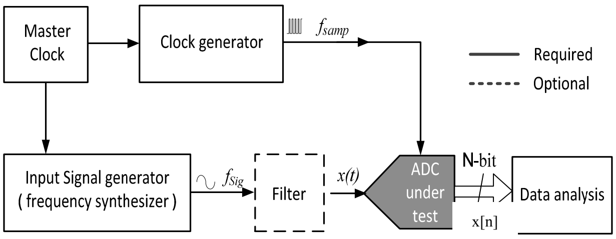

Fig. 1: Ideal DFT based test setup.

$$J = M\frac{f_{Sig}}{f_{Samp}} = J_{\text{int}} + \delta \quad (3)$$

The sampling is said to be coherent if J in (3) is an integer ($\delta = 0$) that is co-prime with M and non-coherent if J is not an integer ($\delta \neq 0$).

Let $x(t)$ in (4) be the time domain representation of pure analog input signal to ADC.

$$x(t) = V_{OS} + A\cos\left(2\pi f_{Sig} t\right) \quad (4)$$

where, V_{OS} is DC value and A is the amplitude of signal $\boldsymbol{x}$.

Let $x[n]$ in (5) be the analog interpretation of the n^{th} digital output of ADC whose gain error has been calibrated.

$$\begin{aligned} x[n] = V_{OS} + A\cos\left(\frac{2\pi J}{M}n + \phi\right) \\ + \sum_{h=2}^{H} A_h \cos\left(\frac{2\pi hJ}{M}n + \phi_h\right) + w[n] \end{aligned} \quad (5)$$

for n = 0,1,2,….,M-1. M is usually selected to be a power of 2 for faster processing of Fast Fourier Transform (FFT). H is the total number of harmonics present in $x[n]$, ϕ is the initial phase of fundamental in $x[n]$, A_h and ϕ_h are the amplitude and initial phase of h^{th} harmonic respectively such that $A_h << A$ and $\phi_h \in [0,2\pi)$ for all $2 \leq h \leq H$. $w[n]$ corresponds to noise in n^{th} sample. The harmonics in (5) correspond to the distortion of ADC.

The spectral parameters are obtained by taking DFT of M coherently sampled data points. DFT of x[n] is given by (6) and Fig. 2 (Blue) shows the spectrum obtained using coherent sampling.

$$X_k = \frac{1}{M}\sum_{n=0}^{M-1} x[n]e^{-j\frac{2\pi k}{M}n}, \quad for \;\; k = 0,1,2,..,M-1 \quad (6)$$

where k represents the frequency bin's index. With coherent sampling, $k = h*J$ represents the frequency bin of h^{th} harmonic. X_0 corresponds to the DC component in signal $\boldsymbol{x}$. Other values of k correspond to noise. FFT algorithm is used to perform DFT,

From equations (5) and (6), it can be said that, for coherent sampling, X_J and X_{hJ} are given by (7) [12].

$$X_J = \frac{A}{2}e^{j\phi} \;\; \text{and} \;\; X_{hJ} = \frac{A_h}{2}e^{j\phi_h} \quad (7)$$

For other values of k, X_k represents noise. The power of fundamental, h^{th} harmonic and noise can be accurately estimated as P_1, P_h and P_{noise} respectively using (8),

$$\begin{aligned} P_1 = 2\left|X_J\right|^2 = \frac{A^2}{2}; P_h = 2\left|X_{hJ}\right|^2 = \frac{A_h^2}{2}; \\ P_{noise} = \sum_{\substack{k=1 \\ k\neq J,\text{hJ} \\ \text{h}=2,3,..,\text{H}}}^{M-1} \left|X_k\right|^2 \end{aligned} \quad (8)$$

From (8), the spectral parameters such as THD, SNR, SNDR, ENOB and SFDR for a coherently sampled signal can be calculated using (9).

$$\left.\begin{aligned} THD = \frac{\sum_{h=2}^{H} P_h}{P_1}; \quad SNR = \frac{P_1}{P_{noise}}; \quad SNDR = \frac{P_1}{\sum_{h=2}^{H} P_h + P_{noise}}; \\ SFDR = \frac{P_1}{2*\max\limits_{\substack{k=1,..,(M/2) \\ k\neq J}}\left(\left|X_k\right|^2\right)}; \quad ENOB = \frac{SNDR - 1.76}{6.02} \end{aligned}\right\} \quad (9)$$

Multi-tone test: The input signal containing K tones with M sampled points can be given as (10-11).

$$x(t) = \sum_{i=1}^{K} a_i \cos\left(2\pi f_{Sig,i} t\right) \quad (10)$$

where a_i and $f_{Sig,i}$ are the amplitude and frequency of i^{th} frequency tone respectively, such that (11) is satisfied and for ideal DFT test, J_i is an integer for all $i = 1,2,..,K$.

$$J_i = M\frac{f_{Sig,i}}{f_{Samp}} \quad (11)$$

In multi-tone testing, intermodulation distortion (IMD) occurs due to ADC nonlinearities. The intermodulation frequencies may occur at sum and difference frequencies for all possible integer multiples of input frequency tones such as ($2f_{Sig,1}$ - $f_{Sig,2}$) or ($f_{Sig,2}$ - $f_{Sig,1}$), etc. IMD is measured by estimating the power of bin that corresponds to the intermodulation frequency in the spectrum. If the bin corresponding to an intermodulation component is given as m, the power of that frequency component is given by (12)

$$P_{IM} = 2\left|X_m\right|^2 \tag{12}$$

Full spectrum test refers to the ability of a method to provide accurate power of each frequency component in the spectrum. It can also be seen from Fig. 2 (Blue plot) that *ideal DFT based spectral test* method performs Full Spectrum test.

In this article the test procedure for single-tone testing is described.

Test Procedure

After satisfying all conditions mentioned above, the procedure to perform test is as follows.

i) *Acquire M samples*

ii) *Perform DFT on acquired data*

iii) *Take DFT and obtain spectral parameters.*

Summary of Ideal DFT based Spectral test Method

Ideal DFT based spectral test method is the golden reference. If such a test setup is available, it is always recommended to use the method for accurate results. The method is *Universal* (independent of ADC resolution), can perform Full spectrum test and multi-tone test and is also fast. However, it cannot be used when the conditions on test setup are not satisfied.

3. Accurate Spectral Testing with Relaxed Constraints

The spectrum obtained using coherent sampling is shown in Fig. 2 (blue). It can be seen that the spectrum is clean and accurate estimates of THD and SFDR can be estimated. However, achieving coherent sampling is one of the most challenging constraints for ideal DFT based test.

If coherent sampling is not obtained ($\delta \neq 0$), the spectrum could contain huge leakage as shown in Fig 2 (red), thus providing inaccurate results.

In precise amplitude control is not attained (such as in BIST ADCs), the input signal might exceed the ADC input range and result in clipped ADC output. Since clipping introduces non-linearities, the FFT of such data shows large power in several frequency bins as shown in Fig. 2 (magenta), thus providing inaccurate results.

In this section, four methods are presented that relax one or both these conditions. Such methods help faster spectral test (quick test setup) and make BIST ADCs and on-chip spectral testing practical. It should be noted that the test setup still needs to satisfy the other two conditions (Pure input and less jitter).

Consider an N-bit ADC as the DUT with input voltage range [F_b F_t], where any input voltage below F_b is clipped at code 0 and above F_t is clipped at code (2^N-1). Let the input and sampled output of ADC be given by (4) and (5) respectively. A description

I. WINDOWING

Using windows is one of the most widely recommended methods in both industry and academia to perform spectral test when the data is not coherently sampled [4].

Working Principle

Let the input and sampled output of ADC be given by (4) and (5) respectively. Since the condition on coherent sampling is relaxed, J in (3) no longer needs to be an integer. The non-repetitive time domain data is made repetitive by multiplying the data with a window function. This reduces leakage in the spectrum.

An L^{th}-order cosine window, $\boldsymbol{w}$, given by (13) is chosen such that the power of secondary lobes in window is below the noise floor of DUT's spectrum. Once a window is chosen, the non-coherently sampled data is multiplied with $\boldsymbol{w}$, to obtain windowed data $\boldsymbol{wd}$ as given by (14).

$$w[n] = \sum_{m=0}^{L} (-1)^m W_m \cos\left(\frac{2\pi}{M} mn\right), n = 0,1,..,M-1 \tag{13}$$

$$wd[n] = x[n].w[n] \tag{14}$$

Taking DFT of windowed data, $\boldsymbol{wd}$, reduces leakage in the spectrum, but splits each spectral line into several lines, the number depending on the window chosen. This split is due to the presence of primary lobe in the DFT of window which is spread about several bins as shown in Fig. 3.

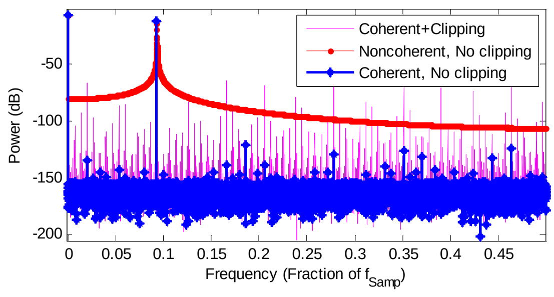

Fig. 2: Spectrums of coherently sampled (Blue), non-coherently sampled (Red) and clipped (Magenta) data.

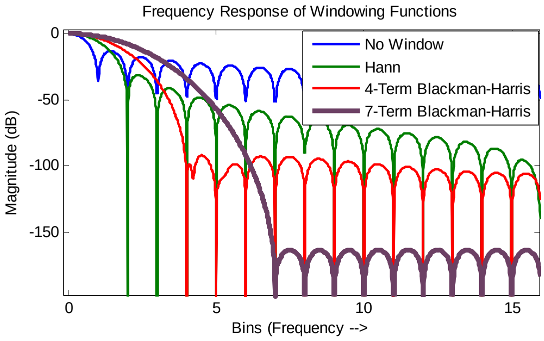

Fig. 3: DFT of different window functions showing primary lobe and the power of secondary lobes.

Therefore, the power of each frequency component is obtained by adding several bins adjacent to the bin closest to the frequency component. The number of bins to be added depends on the chosen window.

Test Procedure

i) *Collect M samples*

ii) *Generate window function*

iii) *Note number of bins in primary lobe.*

iv) *Generate windowed data*

v) *Take DFT of windowed data and estimate spectral parameters*

II. FOUR PARAMETER SINE FIT (FPSF)

Four Parameter Sine Fit (FPSF) method is another approach that is widely used for spectral test. It can be used when the data is non-coherently sampled [4] or clipped, thus, relaxing two conditions for spectral test (coherent sampling and in-range amplitude).

Working Principle

In (5), the ADC output contains not only the fundamental but also harmonics and noise. In this method, the spectral parameters are obtained by estimating amplitudes of frequency components from time domain data.

To maximize accuracy, the data record is first truncated so that it has approximately an integer number of periods of input signal. This truncated record, $\boldsymbol{x_T}$ given by (15), is used to estimate amplitudes of frequency components. In (15), the power of harmonics is neglected since $A_h \ll A$ in high resolution ADCs.

$$x_T[n_T] \approx V_{OS} + A\cos\left(\frac{2\pi J}{M} n_T + \phi\right),\ n_T = 0,1,2,\ldots,T-1 \quad (15)$$

$$= V_{OS} + B\cos\left(\frac{2\pi J}{M} n_T\right) - C\sin\left(\frac{2\pi J}{M} n_T\right) \quad (16)$$

such that, $A = \sqrt{B^2 + C^2}$ (17a); $\phi = \tan^{-1}(C/B)$ (17b)

Equations (15) and (16) are equivalent and from (16), there are four unknowns, namely, V_{OS}, B, J and C. Of the four parameters, J is non-linearly related while the other three parameters are linearly related. T equations can be obtained from (16) if output is not clipped and the four parameters can be estimated using non-linear least squares on T equations. From the final estimates, the amplitude of fundamental can be obtained using (17a). The fundamental component and offset is subtracted from $\boldsymbol{x_T}$ to obtain residue, $\boldsymbol{x_R}$ given by (18).

$$x_R[n_T] = \sum_{h=2}^{H} A_h \cos\left(\frac{2\pi hJ}{M} n_T + \phi_h\right) + w[n_T] \quad (18)$$

$$= \sum_{h=2}^{H} B_h \cos\left(\frac{2\pi hJ}{M} n_T\right) - C_h \sin\left(\frac{2\pi hJ}{M} n_T\right) + w[n_T] \quad (19)$$

such that, $A_h = \sqrt{B_h^2 + C_h^2}$ (20a); $\phi_h = \tan^{-1}(C_h / B_h)$ (20b)

The amplitude of h^{th} harmonic is obtained by estimating B_h and C_h in (19) using linear least squares on T equations (neglect noise). All estimated harmonics are then subtracted from $\boldsymbol{x_R}$ to obtain noise power. Spectral parameters are obtained using the estimated amplitudes of frequency components, noise power and substituting in equations (8-9).

This method can also be used when a signal is clipped and non-coherently sampled. The only change in above method would be to replace n_T with nc_T as given by (21). nc_T contains indices of points that are not clipped in $\boldsymbol{x_T}$.

$$nc_T = \{ n_T \mid F_b < x[n_T] < F_t \} \quad (21)$$

Test Procedure

i) *Acquire M samples and truncate to contain integer cycles*

ii) *Obtain initial estimates of J, B, C,* V_{OS}

iii) *Obtain final estimates of J, B, C,* V_{OS} *using nonlinear least squares.*

iv) *Remove DC and fundamental from initial data to obtain residue*

v) *Perform linear least squares to estimate harmonics*

vi) *Remove harmonics in residue to obtain noise power*

vii) *Calculate spectral parameters using estimates*

III. FUNDAMENTAL IDENTIFICATION AND REPLACEMENT (FIRE)

The FIRE method was recently proposed to provide accurate and robust spectral results for any value of δ, thus eliminating the need for coherent sampling [12].

Working Principle

When a signal is not coherently sampled, the leakage in spectrum is mainly due to the non-coherently sampled fundamental as shown in Fig. 4. The black plot with leakage is the spectrum of a non-coherently sampled data while the red plot is the spectrum obtained using the same data after removing the non-coherent fundamental. It can be seen that leakage is eliminated in the red spectrum. So, FIRE method includes accurately identifying the non-coherent fundamental from data, and replacing it with a fundamental component that has same amplitude and phase but a frequency component that is coherent with respect to the sampling clock.

The time domain data, **x**, is converted to frequency domain data, $\boldsymbol{X_k}$, by performing DFT as given in (6). The fundamental is identified by estimating A, J_{int}, δ and ϕ in (5). The value of J_{int} is obtained by taking the bin that contains the maximum power in spectrum given by (22) (exclude DC). The initial estimates of δ, A and ϕ can be given by (23-25) [12]. With A_0, ϕ_0 and δ_0, accurate estimates can be obtained by performing Newton method (non-linear least squares) on only six equations [12]. Hence, the noncoherent fundamental is accurately identified using frequency domain data and is given by ***x_nc*** in (26), where $\hat{A}, \hat{\delta}$ *and* $\hat{\phi}$, are final estimates of A, δ and ϕ.

$$\hat{J}_{\text{int}} = \arg\max_{1\le k\le M/2} |X_k| \tag{22}$$

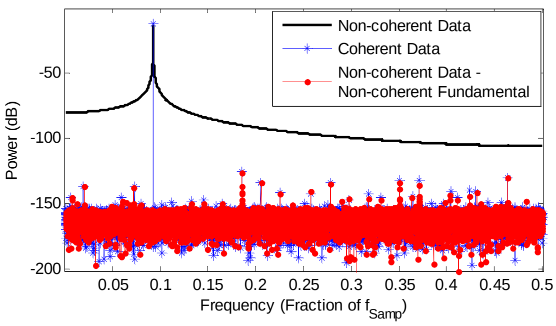

Fig.4 Spectrums of non-coherently sampled data before (black) and after (red) removing the non-coherent fundamental. Blue plot is the reference spectrum of coherently sampled data.

$$\delta_0 = -\frac{M}{2\pi} imag\left(\ln\left(\frac{\frac{X_{J_{\text{int}}}}{X_{J_{\text{int}}+1}} - \frac{X_{J_{\text{int}}}}{X_{J_{\text{int}}-1}}}{\frac{X_{J_{\text{int}}}}{X_{J_{\text{int}}+1}} - \frac{X_{J_{\text{int}}}}{X_{J_{\text{int}}-1}} + e^{j\frac{2\pi}{M}} - e^{-j\frac{2\pi}{M}}} \right)\right) \tag{23}$$

$$A_0 = 2M\left|X_{J_{\text{int}}}\right| \left| \frac{1-e^{j\frac{2\pi\delta_0}{M}}}{1-e^{j2\pi\delta_0}} \right| \tag{24}$$

$$\phi_0 = -imag\left(\ln\left(\frac{2MX_{J_{\text{int}}}}{A_0} \frac{1-e^{j\frac{2\pi\delta_0}{M}}}{1-e^{j2\pi\delta_0}} \right)\right) \tag{25}$$

$$x\_nc[n] = \hat{A}\cos\left(\frac{2\pi\left(J_{\text{int}} + \hat{\delta}\right)}{M} n + \hat{\phi} \right) \tag{26}$$

The estimated non-coherent fundamental is then replaced by a component that has the same amplitude and phase as initial fundamental but has a frequency that is coherently sampled to give $\boldsymbol{x_{New}}$ as shown in (27). The spectral parameters are accurately estimated by taking DFT of $\boldsymbol{x_{New}}$.

$$x_{New}[n] = x[n] - x\_nc[n] + \hat{A}\cos\left(\frac{2\pi\left(J_{\text{int}}\right)}{M} n + \hat{\phi} \right) \tag{27}$$

Test Procedure

i. *Acquire M points and take DFT.*

ii. *Obtain initial estimates of J, A, ϕ.*

iii. *Using newton method, estimate final values of J, A, ϕ.*

iv. *Replace non-coherent fundamental with coherently sampled fundamental to obtain* $\boldsymbol{x_{New}}$.

v. *Take DFT of* $\boldsymbol{x_{New}}$ *and estimate all spectral parameters.*

IV. FUNDAMENTAL ESTIMATION, REMOVAL AND RESIDUE INTERPOLATION (FERARI)

Recently, another method that performs accurate and robust spectral test with simultaneous non-coherent sampling and amplitude clipping was presented, thus relaxing the constraints on Sampling and Amplitude [13].

Working Principle

In this case, the amplitude of input can be slightly greater than the input range of ADC and *J* need not be an integer.. The spectrum of such raw data would have higher harmonic powers and severe leakage due to the over-ranged and non-coherently sampled fundamental. So, it is important to accurately estimate the fundamental and remove it from the original data.

From (5), it is required to evaluate V_{OS}, J, A and ϕ to estimate the fundamental. The amplitude, A, and offset, V_{OS}, are estimated using equations (28-29) from time domain data, where K_t and K_b are the total number of hits on code (2^N-1) and code 0 respectively in **x** (5). The initial estimates of phase, ϕ, and frequency, J, are obtained using (22, 23, 25) from DFT. The final accurate values of ϕ and J are obtained using the initial estimates and performing least squares on set of points that are close to mid-range code [13]. The estimated fundamental is then removed from the raw data and the residue at each code in **x** is stored in a Look up Table (LUT).

$$A = \frac{F_t - F_b}{\cos\left(\frac{K_t - 1}{M}\pi\right) + \cos\left(\frac{K_b - 1}{M}\pi\right)} \tag{28}$$

$$V_{OS} = \frac{F_t + F_b + A\left\{\cos\left(\frac{K_b - 1}{M}\pi\right) - \cos\left(\frac{K_t - 1}{M}\pi\right)\right\}}{2} \quad (29)$$

A coherently sampled signal, $\boldsymbol{x_{c1}}$, with peak-to-peak voltage close to the ADC's input range and frequency close to that of estimated fundamental is generated. The information of harmonics and noise on each sampled point of $\boldsymbol{x_{c1}}$ is given as $\boldsymbol{e}$, and is obtained by interpolating the residue on adjacent codes hit in $\boldsymbol{x}$ using generated LUT. This information of harmonics and noise, $\boldsymbol{e}$, is added to $\boldsymbol{x_{c1}}$, to get $\boldsymbol{x_{Final}}$. DFT is performed on $\boldsymbol{x_{Final}}$ to estimate spectral parameters.

Test Procedure

i) *Acquire M samples*

ii) *Estimate A, V_{OS}*

iii) *Take DFT of data and get initial estimates of J, ϕ*

iv) *Obtain accurate estimates of J, ϕ using least squares*

v) *Remove estimated fundamental from data to obtain residue. Prepare LUT with residue vs. ADC code*

vi) *Generate coherently sampled signal with amplitude equal to ADC's full range, $\boldsymbol{x_{c1}}$*

vii) *Using LUT, interpolate residue onto each code in $\boldsymbol{x_{c1}}$ to get information of harmonics and noise. Add this information to $\boldsymbol{x_{c1}}$ to get $\boldsymbol{x_{Final}}$*

viii) *Perform DFT on $\boldsymbol{x_{Final}}$ and estimate spectral parameters*

4. Comparative Study

In this section, a comparative study on four methods is presented based on different criteria related to spectral test. The comparison is done with a default condition that the data is non-coherently sampled.

Computation time

Computation time complexity for each method is presented based on the major steps in *Test Procedure* section.

1. Windowing: The time complexity of windowing can be given as $O(M\log_2 M)$ as evaluation of DFT is the major time consuming step in the Test Procedure.

2. FPSF: The method involves estimation of parameters using non-linear and linear least squares method, which typically has a time complexity of $O(G^2T)$, where G is the number of parameters and T is the number of equations. The method is highly computation intensive as non-linear least squares is performed until convergence is achieved to estimate the fundamental and linear least squares is performed to estimate each harmonic.

3. FIRE: The time complexity of FIRE can be given as $O(M\log_2 M)$, as the major time consuming step is evaluating the DFT. The time consumed by non-linear least squares method used in FIRE is negligible compared to that of DFT, because it only uses six equations to estimate three unknowns.

4. FERARI: Of all the steps involved in performing this method, evaluating the DFT is the most time consuming process and hence the time complexity can be given as $O(M\log_2 M)$. However, it should be noticed that due to several calculations involved such as least squares and interpolation, FERARI takes considerable amount of computation time.

Table 1 presents the computation time of all four algorithms using non-coherently sampled data. Table 2 provides computation time of FERARI and FPSF methods using non-coherently sampled and clipped data. It can be seen that FIRE and windowing methods have less computation time followed by FERARI and FPSF methods.

Table 1: Computation time of four methods with non-coherent sampling (J = 6537.14, 18-bit ADC, $M=2^{16}$)

Method	Computation Time
Windowing (blackmanharris)	0.003s
FPSF	0.12s (fundamental) 0.83s(20harmonics)
FIRE	0.0028s
FERARI	0.041s

Table 2: Computation time using non-coherently sampled and clipped data (J = 6537.14, Clipping = 2%, 18-bit ADC, $M=2^{16}$)

Method	time
FERARI	0.08s
FPSF	0.14s (fundamental) 1s(20harmonics)

The comparison above is provided for a given data record length. However, if frequency resolution is of interest, windowing requires more data points compared to that of other methods (as explained later). In such cases, windowing consumes more time.

Universal Applicability

A method is said to be *Universal* if it can be readily used without any information about the DUT.

1. Windowing: The primary requirement when using windowing method is that the power of secondary lobes in the chosen window (as in Fig. 3) should be less than that of the noise floor of DUT's spectrum. As a result, not all windows can be used to test all ADCs. Hence, windowing

cannot be termed as a *Universal* method as its application (window selection) depends on resolution of the DUT.

2. FPSF, FIRE and FERARI: All three methods can be termed as Universal methods as they are not dependent on the DUT's information to provide accurate test results.

Frequency Resolution

1. Windowing: When windowing is used for spectral test, each frequency is split into several bins in the spectrum due to the presence of primary lobe. This reduces the frequency resolution that is achievable. From Fig. 3, it can be seen that there is a tradeoff between secondary lobes' power and frequency resolution. The power in secondary lobes is lower for those windows which have more bins in their primary lobe. To test a high resolution ADC, a window with low secondary lobes' power is selected. However, such windows have lower frequency resolution due to presence of more bins in primary lobe. In order to obtain sufficient frequency resolution, the data record length should be increased. This increases data acquisition time and computation time.

2. FPSF: This method only estimates the powers of frequencies that are integral multiples of fundamental as part of test. If there is a particular frequency of interest that is known beforehand, the method can estimate the power of that frequency using least squares. It should be noted that estimating the power of several frequency components using this method becomes highly time consuming and is not preferred if information of several non-harmonic frequencies is needed.

3. FIRE, FERARI: Since clean and accurate spectrums are obtained using both methods, the frequency resolution obtained using FIRE and FERARI is similar to that obtained using ideal DFT based spectral test.

Harmonic Power Calculation

To estimate the power of a frequency component, it is conventional to add power of a set of bins on either side of the bin that is closest to the frequency of interest. When a signal is non-coherently sampled, the bin that is closest to h^{th} harmonic is dependent on δ and is given as $B_{h,a}$ in (30). The accurate power of h^{th} harmonic is obtained by adding power of L bins on either side of $B_{h,a}$ as given in (32).

1. Windowing: In windowing, the value of δ is not known and hence, the estimated bin that is closest to h^{th} harmonic is given as $B_{h,e}$ in (31). The power estimated using windowing is given as $P_{h,e}$ in (32) by adding power of L bins on either side of $B_{h,e}$. It can be seen that if δ is large, windowing calculates harmonic power inaccurately. An example to illustrate this effect is provided using Table 3 and Fig. 5. Fig. 5 shows the spectrum of a 16-bit ADC that is non-coherently sampled and windowed with J = 1031.48 and M = 65536. Table 3 provides the values of $B_{h,a}$ and $B_{h,e}$ with estimated error. It can be seen that windowing provides inaccurate estimate of 12th harmonic power and results in wrong THD, SFDR specifications.

As a result, for large δ, windows method cannot provide accurate harmonic power. However, if δ is small (of the order of 0.01), the method can be used to estimate h^{th} harmonic power accurately.

$$B_{h,a} = round\left(h*\left(J_{\text{int}} + \delta\right)\right) = hJ_{\text{int}} + \text{round}\left(h\delta\right) \qquad (30)$$

$$B_{h,e} = hJ_{\text{int}} \qquad (31)$$

$$P_{h,a} = 2\sum_{i=B_{h,a}-L}^{B_{h,a}+L} \left|X_i\right|^2 \quad (32); \quad P_{h,e} = 2\sum_{i=B_{h,e}-L}^{B_{h,e}+L} \left|X_i\right|^2 \qquad (33)$$

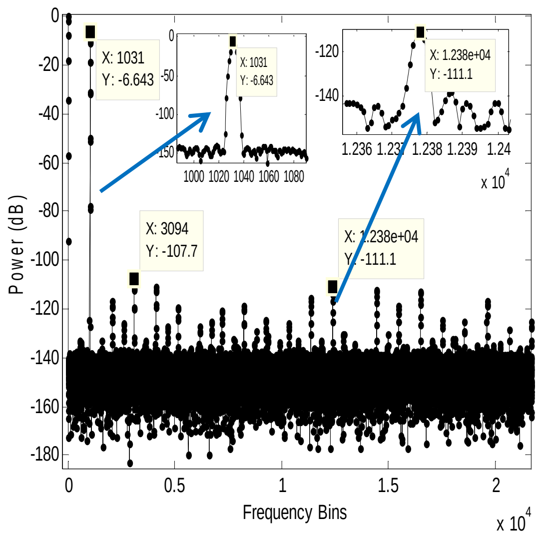

Fig. 5: Spectrum showing the center-bin of fundamental and harmonic lobes of a windowed data.

Table 3: Table showing the error in estimating the center-bin of harmonics using Windows method for J = 1031.48

Harmonic	Actual center bin, $B_{h,a}$	Estimated center-bin using windows, $B_{h,e}$	Error $B_{h,a}$-$B_{h,e}$
1	1031	1031	0
3	3094	3093	1
12	12378	12372	6

2. FPSF: The power of harmonics is accurately estimates using T equations.

3. FIRE: Since the accurate value of δ is known in FIRE, the bin closest to h^{th} harmonic is accurately identified and the power of h^{th} harmonic is calculated correctly.

4. FERARI: The information of harmonics and noise is present in the residue after removing the over-ranged, non-coherent fundamental. Residue interpolation on to each point of the coherent fundamental ensures that the harmonic power is correctly estimated from final DFT.

SNR Calculation:

1. Windowing: Since the primary lobe of the window convolutes with the non-coherently sampled spectrum, windowing modulates the noise power and results in inaccurate SNR values

2. FPSF: If the data only contains fundamental and harmonics apart from noise, this method can provide accurate SNR values. However, if there are other frequency components in the data, the method considers them as noise and provides inaccurate estimate of SNR. To remove such non-harmonic spurs, it is required to know their frequencies beforehand so that their power will not be included in noise power calculation.

3. FIRE: Accurate estimates of SNR are provided using FIRE as the method does not affect noise.

4. FERARI: As the method includes interpolation of residue, the total noise power estimated from the final spectrum will be less than the actual noise power. This is because interpolation smoothens the noise power. However, the estimated error in SNR is less than 1.0 dB.

Multi-tone test

1. Windowing: When the number of tones is less, windowing can perform Multi-tone test on non-coherently sampled data. However, care should be taken such that the lobes around tones are separated and no significant frequency (IMD or Harmonic) components fall in the primary lobes.

2. FPSF: The method cannot be readily used to perform multi-tone test. It is because, when estimating the fundamental, the method assumes other frequency components are of negligible power. This condition is violated in multi-tone test. Though the method can be modified to estimate K tones (K should be known), estimating tones in time domain is very time consuming.

3. FIRE: This method can be repetitively used to perform Multi-tone test provided there are less number of tones and the tones are not close to each other in the spectrum. Modifications can be made to use FIRE method for many-tone test, however, with those modifications it could be termed as a new method.

4. FERARI: The method cannot be readily used to perform multi-tone testing.

So, the *ideal DFT based spectral test* is still considered as the state-of the-art method to perform many-tone test.

Convergence

1. Windowing, FERARI: Both the methods do not have convergence issues as they do not use non-linear estimation.

2. FPSF: As the method includes estimating parameters from a set of non-linear equations, it is necessary for the algorithm to converge to a global minimum. This convergence is strongly dependent on the initial values chosen for the parameters. Hence, special care must be taken to ensure the method converges.

3. FIRE: In this method, non-linear least squares is used to identify the fundamental. As the initial estimates are selected such that they are very close to the actual values, FIRE method has no problem with convergence and always converges to the accurate values.

Full Spectrum test

In order to estimate SFDR, it is required to find the power of maximum spur in the spectrum, be it either a harmonic or a non-harmonic spur. In cases such as time interleaved ADCs, the maximum spur could be non-harmonic as shown in Fig.6. To provide accurate results, in such cases, a test method should be able to perform full spectrum test.

1. Windowing: Windowing can be used to perform Full spectrum test if the bin containing the spur does not coincide with the primary lobe bins.

2. FPSF: This method cannot provide accurate estimate of SFDR if there is a non-harmonic spur that contributes to SFDR. However, if the maximum spur is a harmonic component, this method can be used accurately. Hence, FPSF cannot be used for Full Spectrum test.

3. FIRE, FERARI: Both the methods provide a clean spectrum with information about each frequency component in the spectrum, thus facilitating full spectrum test.

Clipped Data

As mentioned in the test procedures for all the methods, it can be seen that only the FPSF and FERARI methods have the ability to test a data set when it is simultaneously clipped and non-coherently sampled. Windows method cannot be used with clipped data as it is not designed for such applications. FIRE method cannot be used with clipped data as the present method cannot estimate the fundamental accurately when the data is clipped.

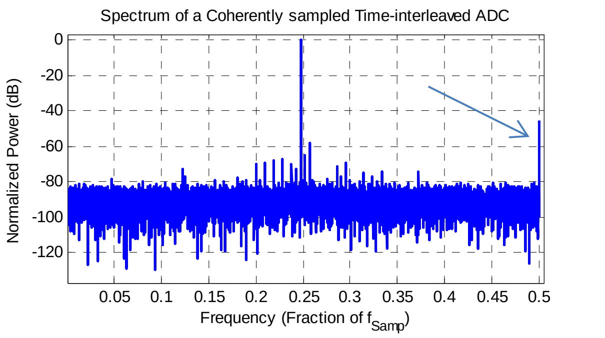

Fig. 6: Spectrum of a time-interleaved ADC showing non-harmonic spur contributing to SFDR.

5. Summary

From the above discussion, it can be stated that, if there is a test setup that satisfies all the challenging requirements for ideal DFT based test, it is recommended to use this test as it is the fastest. Also, it is the only method that can provide accurate spectral results when there are many tones in the signal.

Table 6 provides the summary about situations when a particular method can be used. The default condition is that the data is non-coherently sampled. The first column indicates the test setup requirement or capability. The green box (T) indicates the method can be used and the red box (F) indicates the method cannot be used. The yellow box in Multi-tone test row indicates the methods cannot work robustly, while the yellow box in FERARI column indicates the method can be used, but FIRE method is preferable as it is way faster than FERARI.

TABLE 6: Ability of four methods to provide accurate test results based on Test Setup.

Test Setup	Window	FPSF	FIRE	FERARI
Need fast test	T	F	T	T
Can obtain small δ	T	T	T	T
Cannot control δ	F	T	T	T
Knows DUT Resolution	T	T	T	T
Does not know DUT's Resolution	F	T	T	T
Need Full Spectrum Test	T*	F	T	T
Only Need ENOB and SNDR	T	T	T	T
Need Multi-tone Test	T*	F	T^	F
With Clipped Data	F	T	F	T

*: Provided the bin containing the frequency of interest is not in the primary lobe.

^: Provided the tones are well separated and less number of tones are present in the signal.

Hence, Table 6 provides a means to select a test method based on the available test setup to obtain accurate spectral results.

Scope for Future Work: FIRE method can be slightly modified to accommodate for multi-tone testing without any conditions. Since two of the conditions are relaxed in FERARI method, future work could include relaxing the other two requirements to obtain a spectral test solution that relaxes all the stringent conditions and provides accurate spectral results. Such a method decreases the test setup cost and time and helps design BIST ADCs.

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