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Non Coherent Spectral Analysis of ADC using FFT Windows: an Alternative Approach

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Abstract *- This article presents an alternative method for testing Analog-to-Digital Converters (ADC) by non-coherent spectral analysis using weighting windows and focusing on the maxima of the window spectrum instead of summing the power inside the main lobe of the window spectrum, as it is always done. Comparing to the existing method, theoretical laws for computing fundamental, harmonics and noise power spectral densities are derived and software simulations are used to validate them..*

Keywords *- ADC Test, spectral analysis, coherence, weighting windows*

I. Introduction

ADC test by spectral analysis is a well known and accurate method for the quantization of ADC figures of merit in terms of Signal-to-Noise Ratio (SNR), Spurious Free Dynamic Range (SFDR), Total Harmonic Distortion (THD), Signal-to-Distortion-and-Noise Ratio (SINAD) and the derived Effective Number Of Bits (ENOB). To this purpose, a near full scale sine wave is provided to the device under test (DUT) input, then a Discrete Fourier Transform (DFT) of the output taken data is computed, and, finally, the power spectrum densities of fundamental, harmonic, spurious, and wide band noise components are derived from the obtained spectra [1-3].

This method only requires that the input sine wave to be the purest and that clock signal to have the smallest jitter. In addition, a condition must be verified between input and clock frequencies: the coherence. This condition is formerly known as:

$$\frac{f_{in}}{f_s} = \frac{m}{N} \qquad (1)$$

Where are respectively the input sine wave signal frequency, the sampling clock frequency, the number of periods of the sine wave, and the number of samples in the data acquisition. There is a double condition in equation (1): first, the number *m* of input sine periods in the acquisition should be an integer. This condition is required because of the fact that FFT processing applies to a finite length data output stream, assuming this stream is periodic, repeating itself to infinite time. Having a integer number of periods of the input sine wave in the output data stream avoids any discontinuity between the last sample of one motif and the first sample of the following one. In the spectral domain, discontinuities make the energy of a component spread over the entire spectrum. This is well known as "spectral leakage" [4].

Secondly, it is necessary to obtain all the ADC output codes in the output data stream, so m and N must be relatively prime. As N is chosen to be a power of two for FFT computation, m must be and odd integer.

In real ADC testing, coherence cannot be precisely obtained because of the lack of precision in the frequencies delivered by waveform generators. So a non negligible part of spectral leakage occurs and a good solution is the use of weighting windows, whose particular time domain shape reduce the discontinuities at the bounds of the measurement time window, and reduce spectral leakage. The second section of the paper derives the common method based on the energy of the main lobe of the window transform to compute the components PSD. The main disadvantages of this method are explained. Third paragraph talks about the proposed method, based on the window's main lobe maxima and zero padding. A comparison is done, including statistical aspects of the two methods. Finally, the conclusion present possible solutions on how to reduce the variance of the noise power estimation.

II. Non Coherent Testing: Use Of Weighting Windows

Non-coherence in ADC testing is caused primarily by the lack of precision of the waveform generators effective frequencies. First, some generators just have three or four digits of resolution when setting the frequency for the delivered signal. Secondly, the generators are based on local oscillators with jitter and frequency offset. While jitter is responsible for phase noise, frequency offset cause non-coherence.

A precision of about 100 ppm is common for medium quality generators, so for 1 MHz, the frequency offset should be 100 Hz. Fig. 1 and 2 easily show that a 100 Hz offset compared to a 1.0021972656MHz input sine wave (with 10 MHz sampling clock) is catastrophic for spectrum analysis.

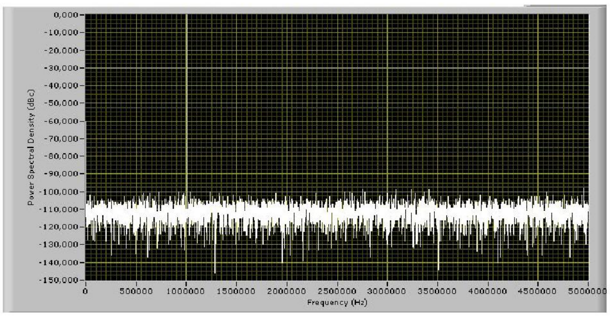

Fig. 1. Acquisition with coherent frequency.

Fig. 1 and 2 are results of simulation of a 12-bit 10 MSPS A/D Converter, with our test software, written in LabVIEW®. Fig. 2 shows not only that a bias is made on the fundamental PSD estimation but also that estimation of harmonics and noise power is not possible.

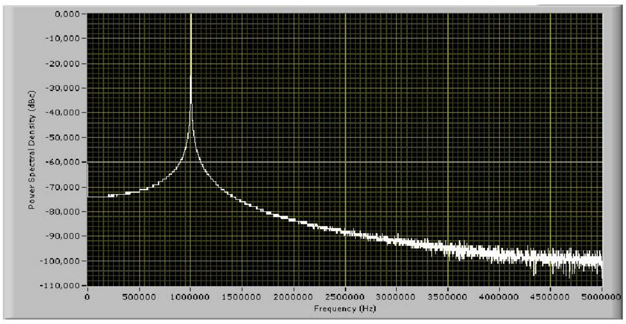

Fig. 2. Acquisition with coherent frequency.

The use of time weighting windows is a good solution to correct this issue. Windows are generally cosine based mathematical functions whose main particularity is to be zeroed at their boundaries. So, if we multiply term by term an N-samples constructed window by the N-samples data acquisition sequence, the widowed data is downed to zero at its boundary, so the so called discontinuities have been greatly attenuated.

Multiplication in the time domain provides convolution in the frequency domain. Besides IEEE standards are not clear from choosing the window, one can say that window needs to have side lobes lower than the ADC noise floor. The classic method in the literature [5][6] says that the energy contained in the side lobes is negligible in front of that of main lobe and one has just to compute the power inside the main lobe of the windowed signal spectrum to obtain the fundamental power.

It is commonly adopted that the main lobe width is about *(2 λ + 1)* bins, where λ is the number of cosine terms of the time window. This is empirical and does not have enough accuracy. Recent works [6] have used Discrete Prolate Spheroidal Sequences (DPSS) class windows that are totally defined by their main lobe width and are more accurate with that. In addition, noise is present all over the frequency axis and it is difficult when computing the power inside the main lobe to subtract the noise part. For those reasons, we are interested with the peak of the main lobe, which is actually the bias brought by the use of windows in spectral analysis. The following explain our investigations.

III. THE PROPOSED METHOD

A. Estimation of Harmonic Components

Now let us consider a discrete-time, noiseless, analog sine wave, at coherent frequency :

$$x(n) = A.\sin\left(2\pi\frac{m}{N}n.f_s\right),\quad \text{with } A \in \mathbb{R}, n = 0,1,...,N-1 \tag{2}$$

If we multiply to x(n) a window sequence w(n), so :

$$x_w(n) = x(n).w(n) \tag{3}$$

Then the discrete time Fourier transform of $X_w(k)$ is :

$$\begin{cases} X_w(k) = \sum_{n=0}^{N-1} x(n)w(n).e^{-j2\pi k\frac{n}{N}}, \\ k = -N/2, -(N/2-1), ..., -1, 0, 1..., N/2-1 \end{cases} \tag{4}$$

Where j is the complex number such that $j^2=-1$ and where k is the spectral bin frequency index.

As $x(n)$ is noiseless, its power spectrum density is composed by only one bin at frequency $m.f_s/N$ (and also its image). The spectrum of $x_w(n)$ is the convolution between the spectrums of $x(n)$ and $w(n)$; and as input frequency is coherent, the maximum of the spectrum of $w(n)$ is shifted by convolution to the position $m.f_s/N$. So the power spectrum density of $x_w(n)$ is also composed by only one bin at the frequency $m.f_s/N$.

And we have:

$$|X_w(m)|^2 = |X(m)|^2.W^2(0) \tag{5}$$

With

$$|X(m)|^2 = A^2/2 \text{ and} \quad W(0) \triangleq \sum_{n=0}^{N-1} w(n)e^0 = \sum_{n=0}^{N-1} w(n) \tag{6}$$

Then an estimation of the power spectral density of the noiseless coherent signal x(n) is:

$$\hat{\sigma}_x^2 = \frac{2}{N.W^2(0)} \left| X_w(m) \right|^2 \qquad (7)$$

When non coherent sampling of the same signal occurs, the frequency of the input signal does not coincide anymore with one of the frequencies of the discrete spectrum, as :

$$f_{in} = (m + \Delta m) f_S / N, 0 < |\Delta m| \leq 0.5 \qquad (8)$$

As a consequence, when convolution operates, the maximum of the shifted window spectrum still coincides with the input signal frequency, but not the discrete spectrum frequencies, as it can be seen on fig. 3.

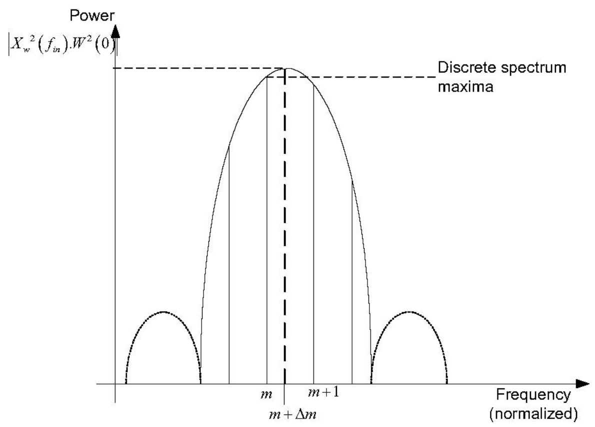

Fig. 3. Non coherent sampling power spectrum (zoom on the main lobe).

The windowed signal power spectrum $|X_w(k)|^2$ is then composed by all the DFT bins, with a maximum on the nearest bin of input frequency, that's to say at the m-th bin. And then,

$$\hat{\sigma}_x^2 = \frac{2}{N.W^2(\Delta m)} \left| X_w(m) \right|^2 \qquad (9)$$

If we could know the magnitude spectrum of the window at any point, we could use this method. But this is not the case. As $W^2(0)$ is known, we can assimilate $W^2(\Delta m)$ to $W^2(0)$ and use relation (7). Fig. 4 shows how much bias is brought by the estimator (9). A 12-bit ideal ADC with full scale input has been simulated, varying input frequency so that the non coherence part Δm is absolutely below 0.5 bin. As the reader can see, for any used window, including rectangular (i.e. no windowing), the bias is negligible (below 0.8 percent, that is below 0.03 dB error on an SNR measurement) on the fundamental power estimation when the non coherence part is below 0.05. In particular, when a 7-term Blackman-Harris window is used, the bias is only 0.4 percent for a non coherence part equal to 0.1 bin.

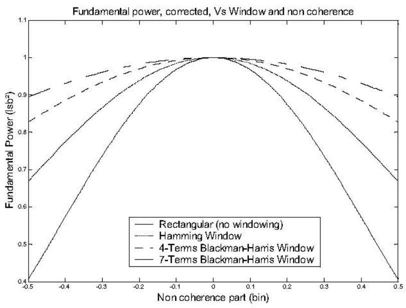

Fig. 4. Bias on estimator vs non-coherence.

So the idea is to decrease the discrete frequency increment $\Delta f = f_s / N$ to reduce the non coherence part.

One solution is to take a larger number of samples. This is not a good idea in industrial context because it multiplies test time. Another solution is the use of zero padding. The principle consists of adding zeroes to the windowed data, so the number of samples is virtually increased, with no additional information. This technique is generally used to make more accurate frequency estimation in the spectral domain. From an original acquisition of N samples, adding $N(ZP - 1)$ zeroes (ZP is the zero padding coefficient) after the windowed signal allow for dividing the frequency increment by ZP. And non coherence part Δm is also divided by ZP.

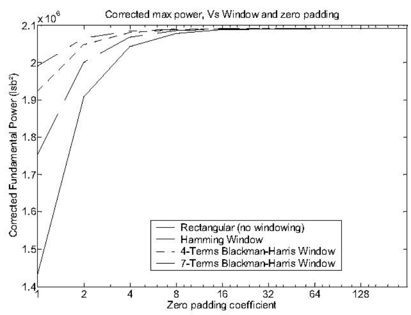

Fig. 5. power of sine input corrected, vs zero padding coefficient and chosen window.

Some software simulations have been realized, with an ideal 12-bit ADC with near full scale input and $\Delta m = 0.333$ bin. Fig. 5 above shows that for any chosen window, having a Zero Padding coefficient as low as 16 allow the recover of a good value of the original power. Using 7-Term Blackman-Harris window help to limit the zero padding coefficient to four, reducing the increase of FFT processing time.

Tables 1 & 2 compile results and errors (relative to theoretical) in percent for simulation of a 12-bit ADC,

$N=16384$ samples and $\Delta m = 0.4$ bin, respectively for 4-term and 7-term Blackman-Harris window.

TABLE I
RESULTS FOR 4-TERM BLACKMAN-HARRIS WINDOW

zero padding	Effective Δm	Power (lsb²)	Power estimation Error (%)
1	0.333	1921990.36763283	8.08326
2	0.1665	2047472.45391083	2.08224
4	0.08325	2080048.70023233	0.52432
8	0.041625	2088269.92937449	0.13115
16	0.0208125	2090330.14867702	0.03263
32	0.01040625	2090845.48191147	0.00798
64	0.005203125	2090974.34715185	0.00182
128	0.002601563	2091006.55819762	0.00028
256	0.001300781	2091014.61425993	-0.00010

Theses tables show that with ZP=8 or 16 for example the results are very close to theoretical $A^2/2$.

TABLE II
RESULTS FOR 7-TERM BLACKMAN-HARRIS WINDOW

zero padding	Effective Δm	Power (lsb²)	Power estimation Error(%)
1	0.333	1989916.62475364	4.83478
2	0.1665	2065281.91016924	1.23053
4	0.08325	2084555.40391623	0.30880
8	0.041625	2089400.89359167	0.077073
16	0.0208125	2090614.12584245	0.01905
32	0.01040625	2090917.47095953	0.00454
64	0.005203125	2090993.34921137	0.00091
128	0.002601563	2091012.30156883	9.48971e-6
256	0.001300781	2091017.04849742	-0.00021

B. Estimation of a Broad Band Noise Power

Let us now consider a N-samples white, Gaussian noise signal $r(n)$ with variance σ_r^2. The windowed noise is noted $r_w(n) = r(n).w(n)$ and its power spectral density is defined by relation(10) :

$$\begin{cases} R_w(k) = \sum_{n=0}^{N-1} r(n)w(n).e^{-j2\pi k\frac{n}{N}}, \\ k = -N/2, -(N/2-1), -1, ..., 0, 1..., N/2-1 \end{cases} \quad (10)$$

An estimation of the noise variance is then :

$$\hat{\sigma}_r^2 = \frac{2}{N^2 NNPG} \sum_{k=0}^{N-1} |R_w(k)|^2 \quad (11)$$

Where *NNPG* is the normalized noise power gain of the window and is defined by :

$$NNPG = \frac{\sum_{n=0}^{N-1} w^2(n)}{N} \quad (12)$$

The purpose is now to express the term *NNPG* as a function of the maxima of the window power spectrum. The Equivalent Noise Bandwidth of the used window can is defined by :

$$ENBW = N \frac{\sum_{n=0}^{N-1} w^2(n)}{\left[\sum_{n=0}^{N-1} w(n)\right]^2} = \frac{N^2 NNPG}{W^2(0)} \quad (13)$$

Then a new expression for *NNPG* is :

$$NNPG = \frac{W^2(0).ENBW}{N^2} \quad (14)$$

Finally, the estimation of noise variance can be re-written by the following relation :

$$\hat{\sigma}_r^2 = \frac{2}{W^2(0).ENBW} \sum_{k=0}^{N-1} |R_w(k)|^2 \quad (15)$$

C. Estimation of Broad Band Noise Components Included in a Read ADC Output Data Signal

The ADC output signal *y(n)* is the sum of a fundamental sine wave *x(n)*, some H harmonics sine waves $x_h(n)$, $h=2,3,...,H+1$, eventually some S spurious sine waves $x_s(n)$, $s=H+2,H+3,...,S+H+1$, and finally the *r(n)* white, Gaussian noise studied above. So the magnitude squared spectrum of *y(n)* is a set of parts which concern deterministic signals corrupted by noise and parts of noise only.

If the total number of samples is *N*, and if we note N_R the number of DFT bins exclusively containing the noise part, we can define N_R by the relation :

$$N_R = N - (2\lambda + 1)(2H + 2S + 1) \quad (16)$$

Where $(2\lambda + 1)$ is the number of bins considered to be in the main lobe of the used window. At this time, the expression of the estimated noise variance is given by :

$$\hat{\sigma}_r^2 = \frac{2}{N_R N.NNPG} \sum_{k \in B_R} |Y_w(k)|^2 \quad (17)$$

Where $Y_w(k)$ is the windowed DFT of the complete ADC output signal and where B_R is the part of the spectrum that only contains noise component. By replacing *NNPG* by its expression in (14), we can write the new expression of $\hat{\sigma}_r^2$:

$$\hat{\sigma}_r^2 = \frac{N}{N_R}\frac{2}{W^2(0).ENBW}\sum_{k\in B_R}\left|Y_w(k)\right|^2 \quad (18)$$

Then for any sinewave $x_i(n), i = 1,2,...,H+S+1$ contained in the ADC output signal, we can write an estimation of its power, like it is when coherent sampling and no windowing occur :

$$\hat{\sigma}_{x_i}^2 = \frac{2}{N.W^2(0)}\left|X_w(m)\right|^2 - \frac{2}{N}\hat{\sigma}_r^2 \quad (19)$$

That is,

$$\hat{\sigma}_{x_i}^2 = \frac{2}{N.W^2(0)}\left|X_w(m)\right|^2 - \frac{2}{N_R}\frac{2}{W^2(0).ENBW}\sum_{k\in B_R}\left|R_w(k)\right|^2 \quad (20)$$

D. Zero Padding Consideration and Choice of the Window for Broadband Noise Power Estimation

When using zero padding (with ZP coefficient), there are *ZP.N* frequency bins and $N_{R\_ZP} = ZP.N_R$ bins of noise.

So the expression of noise variance becomes :

$$\hat{\sigma}_r^2 = \frac{N}{N_R.ZP^2}\frac{2}{W^2(0).ENBW}\sum_{k\in B_R}\left|Y_w(k)\right|^2 \quad (21)$$

Last remark for the choice of the used window: we have seen that any window can be chosen for the estimation of the fundamental power. Any window can be chosen for estimation of a noise only signal. But for the estimation of a noise with the presence of a large bin which is the fundamental, it is evident that noise power estimation requires the side lobes of the window to be small enough. So the side lobe maximal energy must be in accordance with ADC resolution. In all cases, a 7-term Blackman-Harris window seems to work very well.

IV. CONCLUSION

An alternative method for computing non coherent spectral analysis of ADC performance has been presented in this paper. It focuses on zero padding techniques to improve frequency resolution, and obtaining the power of a harmonic component by the maximum of the window main lobe. Noise variance has been re-written as a function of the window parameters W(0) and ENBW. In comparison to the usual main lobe energy method, this one has the advantage of regarding only one bin, and not adding all the noise parts of the bins contained in the main lobe to the harmonic power. In addition, no numbers of bins are needed to make the sum, as a function of the used window. There is still another disadvantage. When increasing the number of samples, the variance of the estimation of the noise power increases too. It may be interesting to associate the method described here with averaged periodograms methods like Bartlett's and Welch's.

V. REFERENCES

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